Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Construction Of New Soliton And Wave Solutions Of Nonlinear Space-Time Fractionalized Partial Differential Equation Via Modified Kudryashov Method

Sajjad Ali
Department of Mathematics and Statistics, Quaid-e-Awam University of Engineering, Science and Technology, 67480, Nawabshah
Sanaullah Dehraj
Department of Mathematics and Statistics, Quaid-e-Awam University of Engineering, Science and Technology, 67480, Nawabshah
Ayaz Ali Siyal
Department of Basic Science and Related studies, Mehran University of Engineering and Technology Jamshoro, Sindh Pakistan
Muzaffar Bashir Arain*
Department of Mathematics and Statistics, Quaid-e-Awam University of Engineering, Science and Technology, 67480, Nawabshah
Email: muzaffararain@quest.edu.pk
Zainab Ali
Department of Mathematics and Statistics, Quaid-e-Awam University of Engineering, Science and Technology, 67480, Nawabshah
Muhammad Hussain Baig
Department of Mathematics and Statistics, Quaid-e-Awam University of Engineering, Science and Technology, 67480, Nawabshah

ABSTRACT

In this article nonlinear space-time fractional Modified KDV (MKDV), and Generalized Boussinesq (GB) equations are analyzed, identifying a variety of physical phenomena, including fluid dynamics, nonlinear optics, shallow water wave transmission, and plasma physics. The well-known generalized closed-form soliton solutions are obtained by utilizing Modified Kudryashov technique. Through the appropriate travelling wave transformations, after the nonlinear fractional equations with partial differentials are moderated to an algebraic system of nonlinear equations, MATLAB is employed to resolve the resulting algebraic equations with the necessary parameters. Transcendental functions are employed to express the consequent closed-form soliton solutions, and graph are illustrate to demonstrate the physical address of the answers that were produced solutions, which represents, Kink, dark, and bright solutions. That's crucial to note that the Modified Kudryashov approach is a great mathematical tool for obtaining soliton solutions of fractional differential equations that are nonlinear and do not require linearization, perturbation, or needless assumptions that could change the problem's physical behavior. Additionally, the obtained generalized soliton solutions will aid in grasping the intricate dynamics of the equations under consideration and are not documented in the literature.

Keywords: Exact Solutions; Modified Kudryashov Method; Solitary Wave Solutions; Space–Time Equations Involving Fractional Partial Differentials; Riemann–Liouville

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Fractional Derivative.

Introduction

In the area of Fractional Calculus (FPDEs) have same concept of classical differential equations in classical calculus and expand it significantly. Instead of dealing only with integer orders of differentiation and integration, FPDEs work with any order, including fractions. Think of them as a broader set of rules governing how integrate and differentiate things when those operations aren't confined to neat whole-number steps. This makes FPDEs a powerful extension of classical calculus. FPDEs are useful over a broad range of engineering and scientific disciplines. Researchers are diving deep into FPDEs and related integral equations precisely because they keep popping up in practical applications. Processing signals, comprehending biological systems, validating intricate engineering designs, managing advanced technology, evaluating financial markets, researching electrochemical reactions, characterizing materials such as viscoelastic polymers, and characterizing diffusion processes that deviate from the norm are just a few of the areas in which they model complex phenomena. The model for gas dynamics [1], optical communication system [2], fluid mechanics [3], characterizing materials like viscoelastic materials [4], Lie symmetry analysis of nonlinear PDEs and conservation theory [5], and describing diffusion mechanisms that deviate from the standard guidelines. Naturally, scientists and engineers around the world are quite interested in this broad significance. More efforts have been made in recent years to find analytical and numerical solutions for ODEs and FPDEs. [6]. Many advanced techniques have recently been presented to obtain an accurate and numerical solution of (FPDEs) [7], resembles the method of homotopy-perturbation [8], Variational iteration technique [9], The extended tanh and conventional tanh techniques [10], The method of the exp function [11], technique of Hirota bilinear [12], the F-expansion [13], The approach of generalized Riccati equations [14], The sub-equation fractional approch [15-16], Modified simple equation method [17-18], Method of residual power series [19-20], Integral transform [21-22], The algebraic direct approach [23], Fractional transform method [24-25], Modified simple method [26-27], q-Homoopy analysis transform method [28-29], The Cole-Hopf transformation method [30], Fractional bell collocation method [31] and so on. Nonlinear evolution equations (NLEEs) admit a broad variety of closed-form wave solutions, such as solitons, breathers, lump waves, and rogue waves (RWs) [32, 33, 34, 35]. Breather solutions are specifically waves that regularly change. Line breathers and the Akhme-diev [36] breather are common examples, Ma's [37] Petviashvili's research on lump waves first discovered numerically, was advanced using the algebraic technique for integrable systems. NLEE lump solutions are unique rational function solutions that are confined in all geographic directions. [38] the first to use the Schrodinger equation to examine rogue waves mathematically. These waves are rational solutions of NLEEs and are particular in both space and time. The department of the surrounding background waves can treble their height. Breather, lump, and rogue wave solutions, which aid in producing nonlinear dynamics in more complex systems, are usually found in higher-dimensional models.

In this article, our aim is to determine new and more general solitonic solutions of two NSTFPDE's, namely the Modified KDV (MKDV) equation and Generalized

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

Boussinesq equation (GBE) in the sense of accordant fractionalized derivative. The application of the Modified Kudryashov technique the current work was innovative because it addressed these fractional models that had not been previously documented in the literature.

The first model consider is MKDV equation is basic nonlinear equation that appears when studying the relativistic wave equation in quantum mechanics. This equation is widely used in field theory, both classical and quantum, and it is an essential tool in many different areas of physics. [39].The model is provided by

$$D_t^{\beta}u - u^2 D_x^{\alpha}u + D_{xxx}^{3\alpha}u = 0; \qquad 0 < \alpha, \beta \leq 1,\ t > 0 \tag{1}$$

The second model consider is Generalized Boussinesq equation is a nonlinear partial differential equation that describes the behavior of waves in fluids, particularly in shallow water[40] .This model is given by:

$$D_t^{\beta}u + uD_x^{\alpha}u + D_{xxx}^{3\alpha}u - aD_{xx}^{2\alpha}u = 0; \qquad 0 < \alpha, \beta \leq 1,\ t > 0 \tag{2}$$

In both model α, β, a, b, and k are random parameters and α, β are an order of the FDE's.

This article orderly as under: In Section 2, we present some common properties of the modified Riemann-Liouville derivative operator. Section 3 describes the Methodology. Section 4 presents the MKDV soliton solutions and GBE are calculated. Section 5 Presents the Results and Discussion and Physically descriptions to some results are expressed and also concluding remarks are point out in Section 6.

Modified Riemann-Liouville Derivative and its Properties

This article describes method the Modified Kudryashov technique to obtain exact soliton solutions involving time and space fractional order derivatives in nonlinear differential equations with the modified Riemann-Liouville derivative and α is order defined by [41]

$$D_t^{\gamma} q(t) = \begin{cases} \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-\zeta)^{-\gamma-1} [q(\zeta) - q(0)] d\zeta, & \text{if } \gamma < 0, \\ \frac{1}{\Gamma(1-\gamma)} \frac{d}{dt} \int_0^t (t-\zeta)^{-\gamma} [q(\zeta) - q(0)] d\zeta, & \text{if } 0 < \gamma < 1, \\ [q^{(\gamma-m)}(t)]^{(m)}, & \text{if } m \leq \gamma < m+1,\ m \geq 1. \end{cases} \tag{3}$$

the Gamma function Γ (*) is:

$$\Gamma(\gamma) = \lim_{m \to \infty} \frac{m! m^{\gamma}}{(\gamma+1)(\gamma+2)\ldots(\gamma+m)} \tag{4}$$

Properties

i) $$D_t^{\gamma} t^r = \frac{\Gamma(1+r)}{\Gamma(r-\gamma+1)} t^{r-\gamma} \tag{5}$$

ii) $$D_t^{\gamma}[q(t)j(t) = j(t)D_t^{\gamma}q(t) + q(t)D_t^{\gamma}j(t) \tag{6}$$

iii) $$D_t^{\gamma} q[j(t)] = \frac{dq(j(t))}{dj(t)} D_t^{\gamma} j(t) \tag{7}$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

1. Methodology

In way to understand the Modified Kudryashov technique, suppose the following (non)linear STFDE twice independent x and t variables in the form

$$\mathrm{p}\left(u, u_x, u_t, u, \ldots, D_t^{\beta} u, D_x^{\alpha} u, \ldots\right) = 0; \qquad 0 < a, \beta \leq 1$$

(8) where the polynomial p is in u and it is different fractional derivatives, u(x,t) is an unidentified function, and $D_t^{\beta} u, D_x^{\alpha} u$ are the fractional derivatives operator of u. and nonlinear terms are concerned with linear terms

Step 1: Let us think about transforming travelling waves as stated:

$$u(\mathrm{x}, \mathrm{t}) = u(\eta), \text{ where } \eta = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} - \frac{\mathrm{ct}^{\beta}}{\Gamma(1+\beta)} \tag{9}$$

Where $k, \mathrm{c} \neq 0$ constants to be resolved later. The soliton wave variable Eq. (9) allows to reduce Eq. (8) into nonlinear ODE for $\mathrm{u} = \mathrm{u}(\eta)$ as,

$$\mathrm{P}(u, \mathrm{k}u', -cu', k^2 u'', \ldots, -cu', \ldots) = 0,$$

(10)

Step 2: If each term in Eq. (10) has derivatives, integrate it under the assumption that the constant for integration is zero.

Step 3: Assume the resolution of (non)linear ODE given in Eq. (10) can be written through a polynomial in $\mathrm{Q}^i(\eta)$ as follow

$$v(\eta) = \sum_{i=0}^{\mathrm{m}} a_i \, \mathrm{Q}^i(\eta), \qquad a_{\mathrm{m}} \neq 0$$

(11)

where the constants ai (I = 0,1,2,...,m) are determine latter:

$$Q(\eta) = \frac{1}{1 \pm De^{\eta}} \quad , \text{ where D is an integration constant}$$

(12)

The $\mathrm{Q}(\eta)$ is the function to the solution of the following Riccati equation of:

$$\mathrm{Q}'(\eta) = \lambda \ln(\mathrm{A}) \left(\mathrm{Q}^2(\eta) - \mathrm{Q}(\eta)\right)$$

(13)

Where A, ($\mathrm{A} \neq 1$) is a random parameter to determine later. Now, positive integer m could be found by assuming the homogeneity equilibrium between highest order derivative and nonlinear expressions that appear in Eq. (10). To the letter, we describe the degree of $u(\eta)$ like $\mathrm{Deg}[u(\eta)] = \mathrm{m}$ that leads to the degree of the other expression such as [42]

$$\mathrm{Deg}\left[\frac{d^p u}{d\eta^p}\right] = \mathrm{m} + \mathrm{p}$$

$$\mathrm{Deg}\left[u^s \left(\frac{d^p u}{d\eta^p}\right)^q\right] = \mathrm{ms} + \mathrm{q}(\mathrm{p} + \mathrm{m})$$

(14)

Step 4: Plugging Eq. (11) with Eq. (12) into the Eq. (10), combining all terms of identical power $\mathrm{Q}(\eta)$, then Eq. (10) is changed within another polynomial in $\mathrm{Q}(\eta)$, equate an algebraic system of equations for ai (i=0,1,2,...,m), with each coefficient of this polynomial equal to zero,

λ, k, a, b and μ, will be obtained.

Step 5: From algebraic system in step 4, we can find constants $a_i (i = 0,1,2 \ldots, \mathrm{m})$, $, A, a, \mathrm{b}$ and λ with the help of MATLAB2023. Since the generalized solution of Nonlinear ODE (12) have been well-known for us, then plugging $a_i (i = 0,1,2 \ldots, \mathrm{m}), k$ and c into the generalized solutions of Eq. (12) into Eq. (11), we get a

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

series of fundamental solutions of the nonlinear evaluation Eq. (8).

Applications of Modified Kudryashov technique

This section's goal to get the precise soliton solutions by using the Modified Kudryashov approach of the (non)linear STFDEs given in Equations (1) and (2).

Fractionalized Modified KDV equation (FMKDV)

We consider soliton wave variable of Eq. (1) and implement the transformation $u(x,t) = u(\eta)$ and $\eta = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} - \frac{ct^{\beta}}{\Gamma(1+\beta)}$, Where $k, c \neq 0$ constants to be resolved latter and the fractional parameter α, β lies $0 < \alpha, \beta \leq 1$. By plugging Eq. (9) in Eq. (1), the equation can be reduced to (non)linear ODE.

$$-cu - \frac{1}{3} ku^3 + k^3 u'' = 0 \quad (15)$$

assume the solution of ODE Eq. (15) is Eq. (11). By using Eq. (12) and Eq. (13) the homogenous balance between

u^3 and u'' is given as $m = 1$

Thus from Eq. (11), we have

$$u(\eta) = a_0 + a_1 Q(\eta) \qquad a_1 \neq 0 \quad (16)$$

$$u^3 = {a_0}^3 + 3{a_0}^2 a_1 Q + 3a_0 {a_1}^2 Q^2 + {a_1}^3 Q^3 \quad (17)$$

By using Eq. (12) and Eq. (16), we get

$$u'' = 2a_1\lambda^2(\ln(A))^2 Q^3 - 3a_1\lambda^2(\ln(A))^2 Q^2 + a_1\lambda^2(\ln(A))^2 Q \quad (18)$$

By plugging Eq. (16) to Eq. (18) into Eq. (15) and by equate coefficients of identical power of Q, we get the subsequent set of algebraic equations for a_0, a_1, k and c:

$$Q^0: -ca_0 - \frac{1}{3}{a_0}^3 k = 0$$
$$Q^1: -ca_1 - k{a_0}^2 a_1 + a_1 \mathrm{k}^3\lambda^2(\ln(A))^2 = 0$$
$$Q^2: -\mathrm{k}a_0{a_1}^2 - 3a_1\lambda^2\mathrm{k}^3(\ln(A))^2 = 0$$
$$Q^3: -\frac{1}{3}{a_1}^3 k + 2a_1\mathrm{k}^3\lambda^2(\ln(A))^2 = 0$$

After solving the above algebraic system, the constants a_0, a_1, c and k can be achieved with the help of MATLAB2023, we have

Set:1

$$\mathrm{K} = 1,\ a_0 = -\frac{6^{\frac{1}{2}}k\lambda\ln(A)}{2},\ a_1 = 6^{\frac{1}{2}}k\lambda\ln(A)\ ,\mathrm{c} = -\frac{\left(k^3\lambda^2\ln(A)\right)^2}{2} \quad (19)$$

Where λ and A are constant parameters, by using Eq. (19) & (12), the equation Eq. (16) can be conveyed as,

$$u_1(\eta) = -\frac{6^{\frac{1}{2}}k\lambda\ln(A)}{2} + 6^{\frac{1}{2}}k\lambda\ln(A)\ \left(\frac{1}{1+e^{\eta}}\right)$$

After some simplification of above equation becomes,

$$u_1(\eta) = -\frac{6^{\frac{1}{2}}k\lambda\ln(A)\ (e^{\eta}-1)}{2+2e^{\eta}} \quad (20)$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Where $\eta = \frac{k\,x^{\alpha}}{\Gamma(1+\alpha)} - \frac{ct^{\beta}}{\Gamma(1+\alpha)}$.

Where $\eta = \frac{k\,x^{\alpha}}{\Gamma(1+\alpha)} + \frac{\left(k^3\lambda^2\ln(A)\right)^2/2\,t^{\beta}}{\Gamma(1+\alpha)}$

Set:2

$K = 1,\ a_0 = \frac{6^{\frac{1}{2}}k\lambda\ln(A)}{2},$ $a_1 = -6^{\frac{1}{2}}k\lambda\ln(A)$, $c = -\frac{\left(k^3\lambda^2\ln(A)\right)^2}{2}$

(21)

Where, λ and A are constant parameters, by using Eq. (21) & (12), the equation Eq. (16) can be converted as,

$u_2(\eta) = \frac{6^{\frac{1}{2}}k\lambda\ln(A)}{2} - 6^{\frac{1}{2}}k\lambda\ln(A)\left(\frac{1}{1+A^{\eta}}\right)$

After some simplification 0f above equation becomes,

$u_2(\eta) = \frac{6^{\frac{1}{2}}k\lambda\ln(A)\,(e^{\eta}-1)}{2+2e^{\eta}}$

(22)

Where, $\eta = \frac{k\,x^{\alpha}}{\Gamma(1+\alpha)} - \frac{ct^{\beta}}{\Gamma(1+\alpha)}$

Where $\eta = \frac{k\,x^{\alpha}}{\Gamma(1+\alpha)} + \frac{\left(k^3\lambda^2\ln(A)\right)^2/2t^{\beta}}{\Gamma(1+\alpha)}$.

Graphical Illustration:

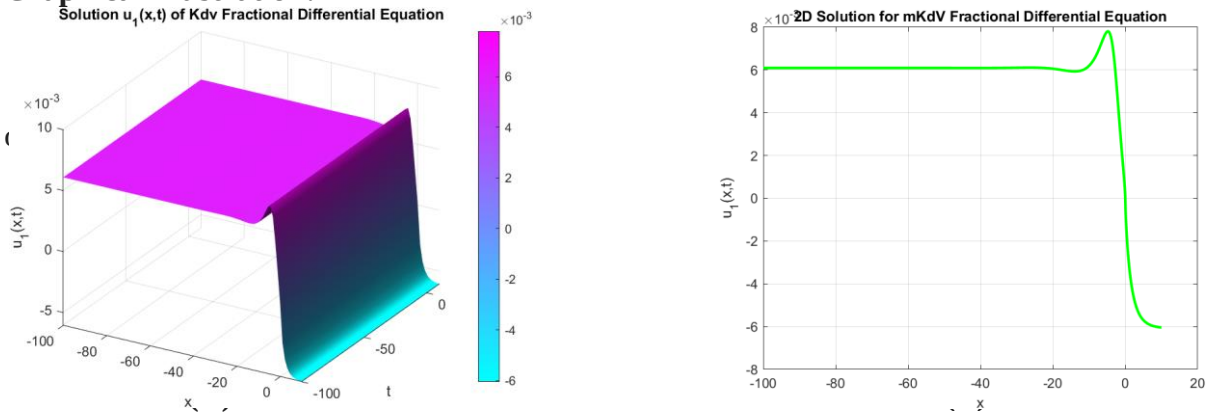


FIGURE1: Wave solution of MKDV with A = 1.01, k = 1, λ = 0.5, α = 0.7, β = 0.4

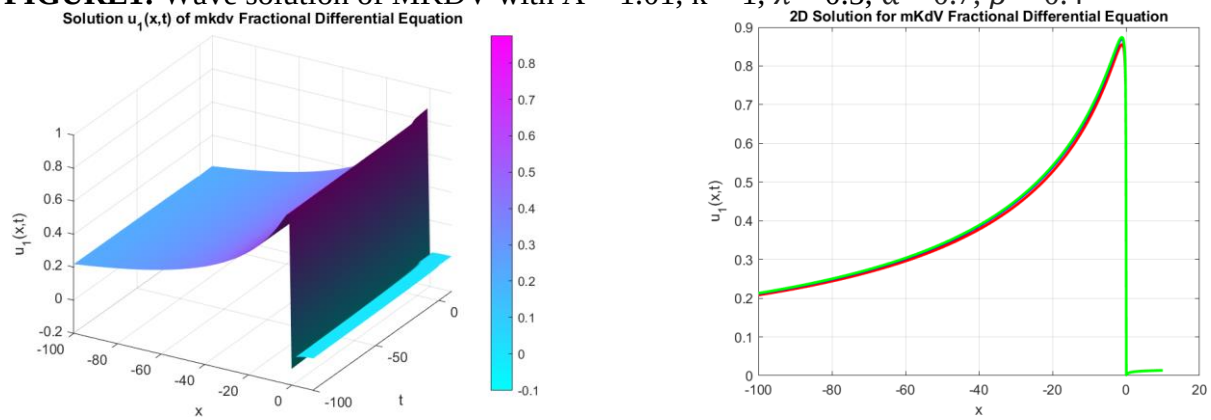
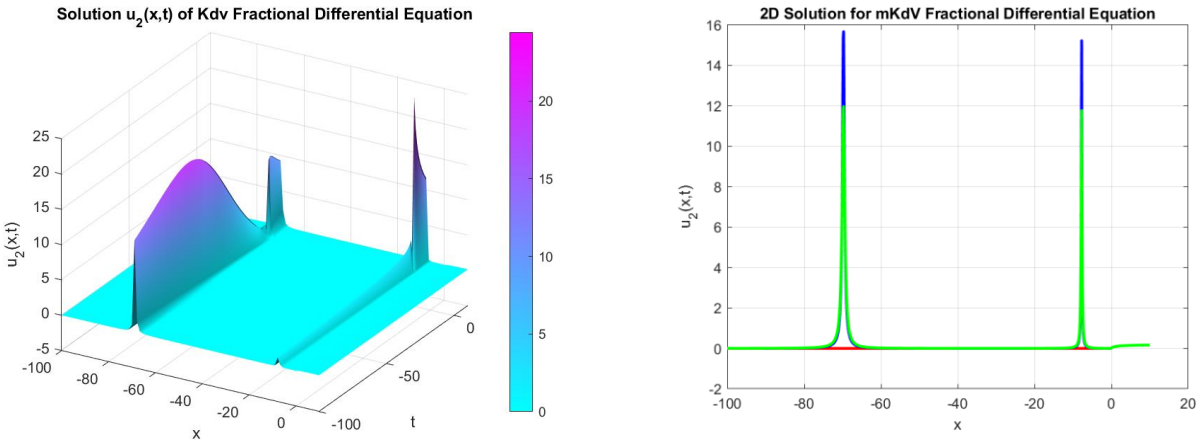


FIGURE2: Wave result of MKDV with A = 0.95, k = 1, λ = 1.5, α = 0.2, β = 0.3

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

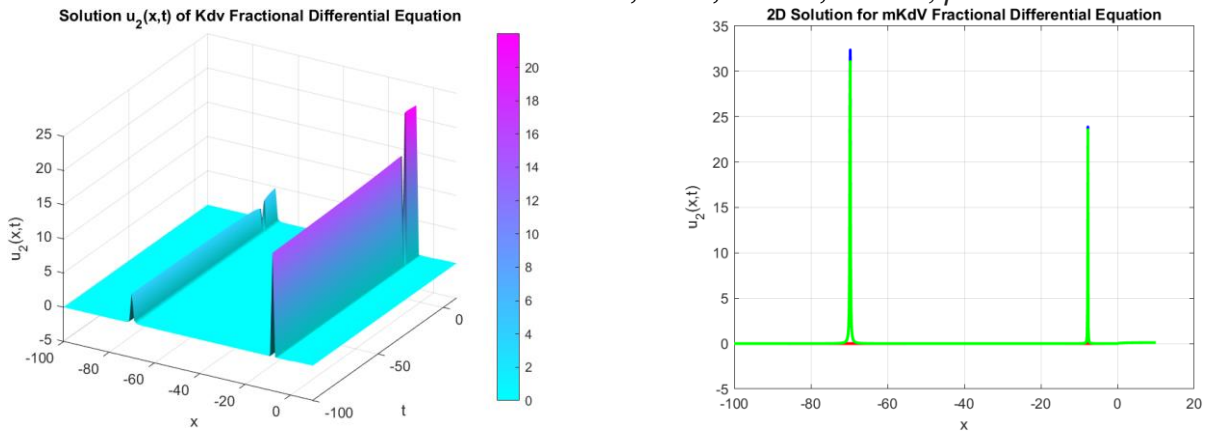
Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about



(A) **(B)**

FIGURE3: Wave result of MKDV with A = 1.5, k = 1, λ = 1.5, α = 0.5, β = 0.4



(A) **(B)**

FIGURE4: Wave result of MKDV with A = 1.1, k = 1, λ = 1, α = 0.5, β = 0.1

Generalized Boussinesq Equation (GBE)

We assume soliton wave variable of Eq. (2)

$u(x,t) = v(\xi)$ and, $\eta = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} - \frac{ct^{\beta}}{\Gamma(1+\beta)}$, Where $k, c \neq 0$ constants and $0 < \alpha, \beta \leq 1$.

Plugging Eq. (9) in Eq. (2), the equation can be reduce to nonlinear ODE. and operate the transformation

$$-cu' - kuu' + k^3u''' - ak^2u'' = 0$$

Integrate above equation w.r.t η

$$-cu + \frac{1}{2}ku^2 + k^3u'' - ak^2u' = 0$$

(23)

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197 Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Suppose the solution of ODE Eq. (23) is Eq. (11). By using Eq. (13) and Eq. (14) the uniform equilibrium between

$u^2$and u'' is $2 + \mathrm{m} = 2\mathrm{m} \Rightarrow m = 2$

Thus from Eq. (11), We've got

$$v(\eta) = a_0 + a_1 Q(\eta) + a_2 Q^2(\eta) \qquad a_2 \neq 0$$

(24) $$v^2(\eta) = {a_0}^2 + {a_1}^2 Q^2(\eta) + {a_2}^2 Q^4(\eta) + 2a_0 a_1 Q(\eta) + 2a_0 a_2 Q^2(\eta) + 2a_1 a_2 Q^3(\eta)$$ (25)

By using Eq. (12) and Eq. (24), we get

$$u'(\eta) = 2a_2 \lambda \ln(A) \qquad \mathrm{Q}^3 + (a_1 \lambda \ln(A) - 2a_2 \ln(A))\, Q^2 - a_1 \lambda \ln(A)\, Q$$ *(26)*

$$u''(\eta) = (6a_2 \lambda^2 \ln(A)^2)\mathrm{Q}^4 + (2\ a_1 \lambda^2 \ln(A)^2 - 10 a_2 \lambda^2 \ln(A)^2)\mathrm{Q}^3 + (3\ a_1 \lambda^2 \ln(A)^2 + 4\ a_2 \lambda^2 \ln(A)^2\ )\ )\mathrm{Q}^2 + a_1 \lambda^2 \ln(A)^2\ \mathrm{Q}$$ *(27)*

By plugging Eq. (24) to Eq (27) into Eq (23) and by equate coefficients of identical power of Q, we get the subsequent set of algebraic equations for a, a_0, a_1, a_2, k and c as follows:

Q^0: $-ca_0 - \frac{1}{2}{a_0}^2 k = 0$,

Q^1: $-ca_1 + ka_0 a_1 + k{a_1}^3 \lambda^2 (\ln(A))^2 + k^2 a{a_1}^2 \lambda \ln(\mathrm{A}) = 0$

Q^2: $-ca_2 + \frac{1}{2}{a_1}^2 k + ka_0 a_2 - 3a_1 \lambda^2 \mathrm{k}^3 (\ln(A))^2 + 4a_2 \lambda^2 \mathrm{k}^3 (\ln(A))^2 - k^2 a a_1 \lambda \ln(\mathrm{A}) + 2a a_2 \mathrm{k}^3 \lambda \ln(\mathrm{A}) = 0$

Q^3: $a_1 a_2 k + 2a_1 \lambda^2 \mathrm{k}^3 (\ln(A))^2 - 4a_2 \lambda^2 \mathrm{k}^3 (\ln(A))^2 - 6a_2 \lambda^2 \mathrm{k}^3 (\ln(A))^2 - 2a a_2 \mathrm{k}^3 \lambda \ln(\mathrm{A}) = 0$

Q^4: $\frac{1}{2}{a_2}^2 k + 2a_2 \lambda^2 \mathrm{k}^3 (\ln(A))^2 + 4a_2 \lambda^2 \mathrm{k}^3 (\ln(A))^2 = 0$

After solving the above algebraic system, the constants a_0, a_1, a_2, c and k can be achieved with the help of MATLAB2023, we have one set of solution

Set:1

For a_0:

$$a_0 = -2\lambda^2 \mathrm{k}^2 (\ln(A))^2 - 2ak\lambda \ln(\mathrm{A})$$

For a_1:

$$\frac{24\lambda^3 \mathrm{k}^3 (\ln(A))^3 + 2a\left(a\sqrt{4\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}\right.}{\left. -8\lambda^3 \mathrm{k}^3 (\ln(A))^3 - 2a\lambda^2 \mathrm{k}^3 (\ln(A))^2 + 5k\lambda \ln(\mathrm{A})\sqrt{\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}\ \right)}{2ak - a + 3ak\lambda \ln(\mathrm{A})}$$

$$-4ak\left(a\sqrt{4\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}\ - 8\lambda^3 \mathrm{k}^3 (\ln(A))^3\right.$$

$$\left. - \frac{-2a\lambda^2 \mathrm{k}^3 (\ln(A))^2 + 5k\lambda \ln(\mathrm{A})\sqrt{\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}}{2ak - a + 3ak\lambda \ln(\mathrm{A})}\ \right)$$

$$+18a\lambda^2 \mathrm{k}^2 (\ln(A))^2 - 4a\lambda^2 \mathrm{k}^3 (\ln(A))^2 + 2\ a\ k\lambda \ln(\mathrm{A}) -$$

$$\frac{6k\lambda \ln(\mathrm{A})\left(a\sqrt{4\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}\ - 8\lambda^3 \mathrm{k}^3 (\ln(A))^3 - 2a\lambda^2 \mathrm{k}^3 (\ln(A))^2 + 5k\lambda \ln(\mathrm{A})\sqrt{\lambda^3 \mathrm{k}^3 (\ln(A))^3 (2ak - a + 4ak\lambda \ln(\mathrm{A}))}\right)}{(a + 5k\lambda \ln(\mathrm{A}))}$$

$$2ak - a + 3ak\lambda \ln(\mathrm{A})$$

For a_2:

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$$a_{2=}\frac{2(a\sqrt{4\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A))}-8\lambda^3k^3(\ln(A))^3-2a\lambda^2k^3(\ln(A))^2+5k\lambda\ln(A)}{2ak-a+3ak\lambda\ln(A)}$$

For c:

$$-\lambda^2k^3(\ln(A))^2-ak^2\lambda\ln(A)$$

Where a, λ, A are constant parameters, put the values of a_0, a_1, a_2 *and* c in equation (24) can be expressed as,

$$u_1(\eta)=-2\lambda^2k^2(\ln(A))^2-2ak\lambda\ln(A)+\frac{-4ak(a\sqrt{4\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A))}-8\lambda^3k^3(\ln(A))^3-2a\lambda^2k^3(\ln(A))^2+5k\lambda\ln(A)}{2ak-a+3ak\lambda\ln(A)}$$

$$+18a\lambda^2k^2(\ln(A))^2-4a\lambda^2k^3(\ln(A))^2+\frac{2}{6k\lambda\ln(A)\left(a\sqrt{4\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A))}-8\lambda^3k^3(\ln(A))^3-2a\lambda^2k^3(\ln(A))^2+5k\lambda\ln(A)\sqrt{\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A)}\right.}$$

$$a\,k\lambda\ln(A)-\frac{(a+5k\lambda\ln(A))\frac{1}{1+A^{\eta}}}{2ak-a+3ak\lambda\ln(A)}$$

$$\left.\frac{2(a\sqrt{4\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A))}-8\lambda^3k^3(\ln(A))^3-2a\lambda^2k^3(\ln(A))^2+5k\lambda\ln(A)\sqrt{\lambda^3k^3(\ln(A))^3(2ak-a+4ak\lambda\ln(A))}}{2ak-a+3ak\lambda\ln(A)}\right)$$

Where, $\eta=\frac{k\,x^{\alpha}}{\Gamma(1+\alpha)}-\frac{ct^{\beta}}{\Gamma(1+\alpha)}$

Graphical Illustration

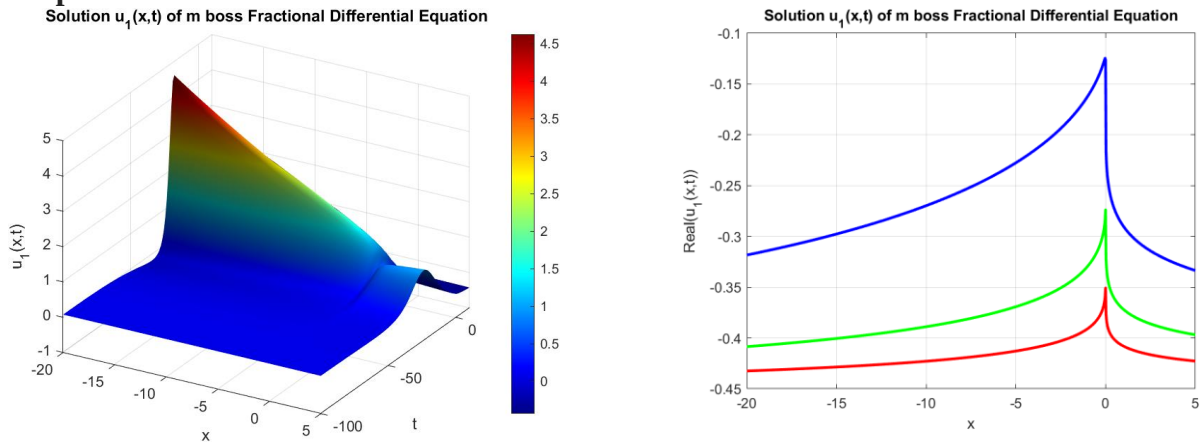


FIGURE5: Wave result of GBE with A = 1.1, a = 1, k = 1, λ = 2, α = 0.2, β = 0.8

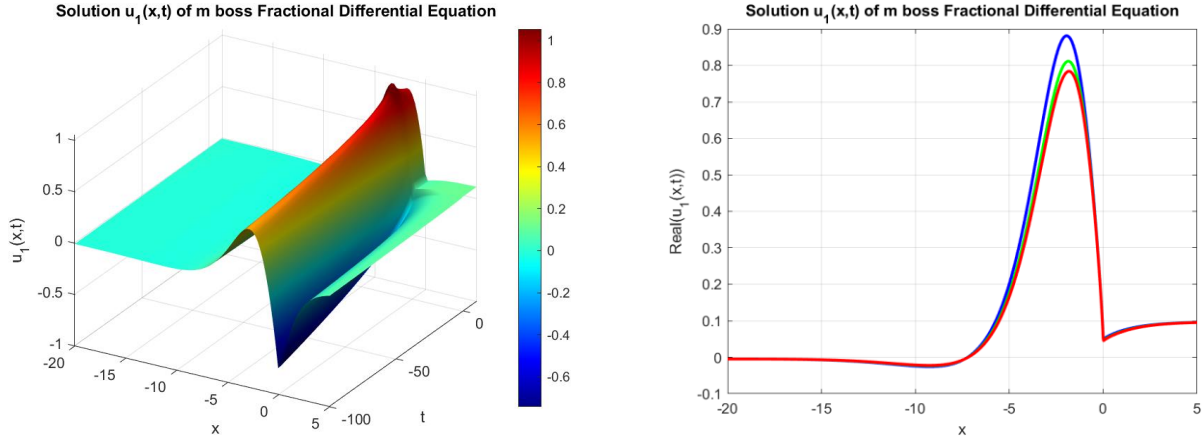
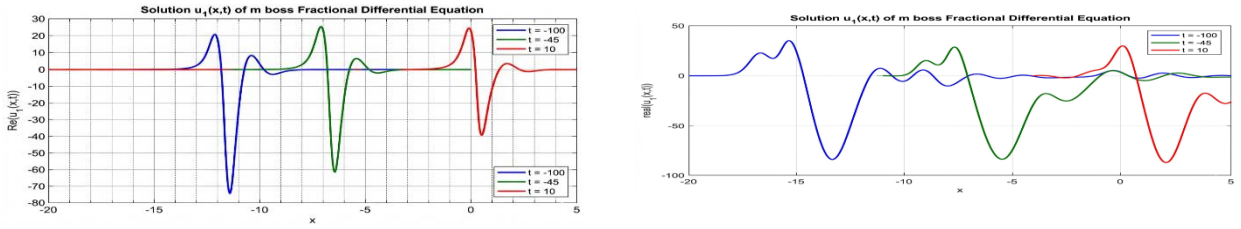


FIGURE6: Wave result of GBE with A = 0.95, a = 1.2, k = 1, λ = 1.5, α = 0.8, β 0.5

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 6 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about



(A) **(B)**

FIGURE7: Wave result of GBE with A = 0.5, a = 1, k = 1, λ = 1, α = 0.8, β = 0.8

(A) **(B)**

FIGURE8: Wave result of GBE with A = 0.5, a = 2.5, k = 1, λ = 2.5, α = 0.5, β = 0.

Numerical Findings and Conversation:

The objective of this part is to demonstrate the several soliton solutions for MKDV and Generalized Boussinesq Equation (GBE) problems graphically. Graphical representation is the effective tool of presentation and interpretation of solutions of mathematical problems in clear and consistent way to have a closer look at the physical behavior of solutions obtained, different values of parameters are chosen after which graphs are plotted. The solutions obtained can be soliton and peak on profiles of taken space-time fractional differential equations. These isolated wave states characterize assorted nonlinear wave phenomena and indicate how the parameters of the system affect the dynamics of the waves signs of velocity of wave determine direction of propagation right motion is present in a positive value of wave speed, and left motion in a negative value. Amplitude and the speed of the wave may be easily adjusted by changing the parameters, which proves the versatility and physical applicability of the model. From Figure 1 and 2 symbolizes the antikink and kink solitons. Figure 3 and Figure 4 represents the peakon solitons respectively, some specific exact solutions to the MKDV equation with certain parameter values, whereas Figure 5 and Figure 6 represents Bright asymmetric solitons with oscillatory respectively. Figure7 and figure 8 represents a singular solitary wave with oscillatory profile, the multiple peaks observed in 2-D plots are not independent solitons but the result of the high frequency dispersive effects, some exact solutions to the Generalized Boussinesq Equation (GBE). The calculations are done via Modified Kudryashov technique. The graphical conspiracy are used to demonstrate the structural characteristics of the results and verify the validity and efficiency of the suggested analytical method.

Conclusions

In this article, we examined two distinct NSTFDE's which arises in wave phenomena named as Modified KDV equation (MKDV) and fractional Generalized Boussinesq Equation (GBE) these models play very crucial role in many fields of sciences and engineering. The two equations are recast using a well-known derivative operator sense as Jumarie modified Riemann-Liouville. Utilizing the Modified Kudryashov method, to obtain the solitary wave solutions in the form of rational and transcendental functions. The range of solitary wave solutions found in this work are the most recent and innovative, and they may be applied in various fields and will

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

greatly aid in knowing the physical behavior of these models. The Modified Kudryashov technique's significance as a quick, efficient, straightforward, and strong method for resolving nonlinear equations in mathematical models and this approach can be applied directly to any kind of problems and requires no linearization, unnecessary presumptions or changes that might alter the physical behavior of the problem..

References:

[1] M. Sadaf, Z. Perveen, G. Akram, U. Habiba, M. Abbas, and H. Emadifar, Correction: Solution of time fractional gas dynamics equation using Elzaki decomposition method with Caputo-Fabrizio fractional deriv ative, PLoS One 20(3) (2025), e0321042. DOI:10.1371/journal.pone.0321042.

[2] M. Vivas-Cortez, M. Sadaf, S. Arshed, K. Rehan, G. Akram, and K. Saeed, Extraction of Optical Soli tons for Conformable Perturbed Gerdjikov–Ivanov Equation via Two Integrating Techniques, Advances in Mathematical Physics 2024(1) (2024), 5936389, DOI:10.1155/2024/5936389.

[3] S. Dehraj , R.A. Malookani, M.B. Arain, N. Nizamani, Oscillating flows of fractionalized second grade fluid with slip effects. Punjab University Journal of Mathematics, 51(11) (2019), 63-75, DOI:10.5402/2012/908386.

[4] S. Reese and M. B¨ol: On a micromechanically-based finite element simulation of the viscoelastic and dam age behaviour of rubber-like polymers, in Constitutive Models for Rubber III, CRC Press, (2026), DOI:10.1007/s10853-005-5058-x.

[5] S. Ray, On conservation laws by Lie symmetry analysis for (2+ 1)-dimensional Bogoyavlen sky–Konopelchenko equation in wave propagation. Computers Mathematics with Applications, 74(6), 1158 1165, DOI:10.1016/j.camwa.2018.03.030.

[6] M.Pakdemirli, Partial Differential Equations Possessing New Complex Derivatives, Punjab University Jour nal of Mathematics, 56(10) (2024), 592-602, DOI:10.52280/pujm.2024.56(10)01.

[7] A. Al-Nana, I. M. Batiha, I. H. Jebril, S. Alkhazaleh, and T. Abdeljawad, Numerical solution of conformable fractional periodic boundary value problems by shifted Jacobi method, International Journal of Mathemati cal, Engineering and Management Sciences 10(1) (2025), 189, DOI:10.33889/IJMEMS.2025.10.1.011.

[8] M. M. AlBaidani, Comparative study of the nonlinear fractional generalized Burger-Fisher equations using the homotopy perturbation transform method and new iterative transform method, Fractal and Fractional 9(6) (2025), 390, DOI:10.3390/fractalfract9060390.

[9] G.Monzon,Fractional variational iteration method for higher-order fractional differential equations, Journal of Fractional Calculus and Applications 15(1) (2024), 1–15, DOI:10.21608/jfca.2023.229366.1027.

[10] S. Ali, A. Ullah, S. F. Aldosary, S. Ahmad, and S. Ahmad, Construction of optical solitary wave solutions and their propagation for Kuralay system using tanh-coth and energy balance method, Results in Physics 59 (2024), 107556, DOI:10.1016/j.rinp.2024.107556.

[11] S. Ramya, K. Krishnakumar, and R. Ilangovane, Exact solutions of time

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

fractional generalized Burg ers–Fisher equation using exp and exponential rational function methods, International Journal of Dynamics and Control 12(1) (2024), 292–302, DOI:10.1007/s40435-023-01267-6.

[12] N. Hemnath, V. Kumar, S. Saha, A. Bhushan, S. Hameed, S. Das and O. Dalvi, Analysis of (3+1) Dimensional MNWI Equation via Hirota's Bilinear Method, Springer Nature, p. 145 (2026), DOI:10.1007/978-981-96-8987-3_11.

[13] M. A. Akbar, N. H. M Ali., The improved F-expansion method with Riccati equation and its applications in mathematical physics. Cogent Mathematics, 4(1) (2017), 1282577, DOI:10.1080/23311835.2017.1282577.

[14] Phoosree, S., Khongnual, N., Sanjun, J., Kammanee, A., & Thadee, W. (2024). Riccati sub-equation method for solving fractional flood wave equation and fractional plasma physics equation. Partial Differential Equations in Applied Mathematics, 100672, DOI:10.1016/j.padiff.2024.100672.

[15] Habiba, U., Salam, M. A., & Khan, K. (2024). Stability, Bifurcation, and Traveling Wave Solutions to the Generalized Time-Fractional Burgers-Huxley Equation. International Journal of Applied and Computational Mathematics, 10(2), 62, DOI:10.1007/s40819-024-01698-5.

[16] Allahyani, S. A., Rehman, H. U., Awan, A. U., Tag-ElDin, E. M., & Hassan, M. U. (2022). Diverse variety of exact solutions for nonlinear Gilson–Pickering equation. *Symmetry*, *14*(10), 2151, DOI:10.3390/sym14102151.

[17] Sheikh, M. A. N., Taher, M. A., Hossain, M. M., & Akter, S. (2023). Variable coefficient exact solution of Sharma–Tasso–Olver model by enhanced modified simple equation method. Partial Differential Equations in Applied Mathematics, 7, 100527, DOI:10.1016/j.padiff.2023.100527.

[18] Cevikel, A. C., & Bekir, A. (2023). Assorted hyperbolic and trigonometric function solutions of fractional equations with conformable derivative in shallow water. International Journal of Modern Physics B, 37(09), 2350084, DOI:10.1016/j.padiff.2023.100527

[19] Al-Smadi, M., Arqub, O. A., & Hadid, S. (2020). An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative. Communications in Theoretical Physics, 72(8), 085001, DOI:10.1088/1572-9494/ab8a29.

[20] Al-Smadi, M., Arqub, O. A., & Hadid, S. (2020). Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method. Physica Scripta, 95(10), 105205 DOI:10.1088/1572-9494/ab8a29.

[21] Abro, K. A., Shaikh, A. A., & Dehraj, S. (2016). Exact solutions on the oscillating plate of Maxwell fluids. Mehran University Research Journal of Engineering & Technology, 35(1), 155-160. Doi: 10.3316/informit.158820205040359.

[22] Akgül, E. K., Akgül, A., & Yavuz, M. (2021). New illustrative applications of integral transforms to financial models with different fractional derivatives. Chaos, Solitons & Fractals, 146, 110877, DOI:10.1016/j.chaos.2021.110877.

[23] Ghayad, M. S., Badra, N. M., Ahmed, H. M., & Rabie, W. B. (2024). Analytic

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

soliton solutions for RKL equation with quadrupled power-law of self-phase modulation using modified extended direct algebraic method. Journal of Optics, 1-13, DOI:10.1007/s12596-023-01624-w.

[24] Asghari, Y., Eslami, M., & Rezazadeh, H. (2023). Exact solutions to the conformable time-fractional discretized mKdv lattice system using the fractional transformation method. Optical and Quantum Electronics, 55(4), 318, DOI:10.1007/s11082-022-04529-3.

[25] Obeidat, N. A., & Bentil, D. E. (2023). Novel technique to investigate the convergence analysis of the tempered fractional natural transform method applied to diffusion equations. Journal of Ocean Engineering and Science, 8(6), 636-646, DOI:10.1016/j.joes.2022.05.014.

[26] Albogami, D., Maturi, D., & Alshehri, H. (2023). Adomian Decomposition Method for Solving Fractional Time-Klein-Gordon Equations Using Maple. Applied Mathematics, 14(6), 411-418, DOI:10.4236/am.2023.146024.

[27] Barikbin, M. S. (2024). Modified simple equation method (MSEM) for solving nonlinear (3+ 1)-dimensional space-time fractional equations. Computational Methods for Differential Equations, 12(2), 413-424, DOI: 10.22034/CMDE.2023.51930.2168

[28] Shah, R., Alkhezi, Y., & Alhamad, K. (2023). An analytical approach to solve the fractional Benney equation using the q-homotopy analysis transform method. Symmetry, 15(3), 669, DOI:10.3390/sym15030669.

[29] Ali, K. K., Maneea, M., & Mohamed, M. S. (2023). Solving nonlinear fractional models in superconductivity using the q-Homotopy analysis transform method. Journal of Mathematics, 2023, 1-23, DOI:10.1155/2023/6647375.

[30] Pirkhedri, A. (2024). Applying Haar-Sinc Spectral Method for Solving time-fractional Burger Equation. Mathematics and Computational Sciences, 5(1), 43-54.

[31] Yüzbaşı, Ş. (2024). Fractional Bell collocation method for solving linear fractional integro-differential equations. Mathematical Sciences, 18(1), 29-40, DOI:10.1007/s40096-022-00482-0.

[32] S. S Kazmi, M. B Riaz, A Jhangeer, Analyzing N-solitons, breathers, and hybrid interactions: comparisons of localized wave dynamics through data points. Nonlinear Dynamics, 113(8) (2025), 8921-8950, DOI:10.1007/s11071-024-10756-y.

[33] M. B. Riaz, A. Jhangeer, S. S. Kazmi, Optical soliton stability in zig-zag optical lattices: comparative analysis through two analytical techniques and phase portraits. Nonlinear Dynamics, 112(24) (2024), 22221 22243, DOI:10.1007/s11071-024-10172-2.

[34] S. S. Kazmi, M. B. Riaz, Comparative analysis of lump, breather, and interaction solutions using a bidirec tional data mapping approach. Scientific Reports, 15(1) (2025), 40242, DOI:10.1038/s41598-025-24067-8.

[35] M. Irshad, M. Hussan, M. Shan, Z. Ali, U. Fatima, I. Jahangir, Exploring Lie Point Symmetries and Exact Solutions for (1+ 1) Dimensional Modified Thomas and (1+ 2) Dimensional Chaffee-Infante Equations. Punjab University Journal of Mathematics, 56(9) (2024), 567-91 DOI:10.52280/pujm.2024.56(9)05.

Online ISSN 3007-3197
Print ISSN 3007-3189

[36] S. F. Tian, H. Q. Zhang, Riemann theta functions periodic wave solutions and rational characteristics for the (1+ 1) dimensional and (2+ 1)-dimensional Ito equation. Chaos,Solitons Fractals 47 (2013), 27–41, DOI:10.1016/j.chaos.2012.12.004.

[37] W.X.Ma.Lumpsolutions to the Kadomtsev–Petviashvili equation. Physics Letters A.379(36) (2015) 1975 8, DOI:10.1016/j.physleta.2015.06.061.

[38] D. H. Peregrine, Water waves, nonlinear Schr¨odinger equa tions and their solutions. ANZIAM J., 25(1) (1983), 16–43, https://doi.org/10.1017/S0334270000003891.

[39] Noor, M. A., Noor, K. I., Waheed, A., & Al-Said, E. A. (2011). Some new solitonary solutions of the modified Benjamin–Bona–Mahony equation. *Computers & Mathematics with Applications*, *62*(4), 2126-2131, DOI:10.1016/j.camwa.2011.06.060.

[40] Kadkhoda, N. (2017). Application of G'G^ 2 G′ G 2-Expansion Method for Solving Fractional Differential Equations. International journal of applied and computational mathematics, 3, 1415-1424, DOI:10.1007/s40819-017-0344-2.

[41] Gepreel, K.A and Omran, S.(2012). Exact solutions for nonlinear partial fractional differential equations. Chinese Physics B 21 (11) 110204, DOI:10.1088/1674-1056/21/11/110204.

- [42] Shang, N. and Zheng, B. (2013). Exact solutions for three fractional partial differential equations by the (G′/G)-method. Laeng International Journal of Applied Mathematics 43 (3): 1-6, IAENG International Journal of Applied Mathematics 43(3).