Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 1 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Thermodynamic's First Law Of Some Black Holes In Einstein's Power Yang-Mills Gravity Using Foliation

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ABSTRACT

In this research article, adopted an approach (Siddiqui et al. 2011) utilizing idea of foliation with virtually displaced hyper-surfaces at outer horizon to build the first law of thermodynamics for some black holes in Einstein's Power Yang-Mills gravity.

Introduction

A link with thermodynamics and black hole begins from a Penrose theory, reverse the entropy rises, showing that when using black holes spacetime. Penrose with Floyd de- veloped a method for the extraction of energy from spinning black holes in 1971 [2]. At that moment when Penrose proposed the argument, it looked impossible to draw energy from a black hole spacetime. Nevertheless, the Kerr black hole spacetime accumulated
energy at the ergo-sphere is cleared earlier. Hence no real energy, moving out of the black hole, tended to lose mass because any suspected mass was contained outside a black hole as just an energy. Ruffini with Christodoulou 1971[3] found out that it has an additional electromagnetic radiation contained inside it due to a mass in the black hole spacetime, for "black hole thermodynamics" as just an electrifying black hole physics branch to develop. The law of energy conservation provides the subject with a solid base. Historically, the relation between heat conduction and chemical transformation is clear. The transfer between heat and mechanical energy is also explained. Later, Maxwell and many others offered a better understanding of macroscopic variables , as the driving factor for microscopic mechanical variables. Thermodynamics classically requires a system's equilibrium state. Therefore, during a vigilant study, the law built an axiomatic basis for thermodynamics and became accepted. Such laws are really a comprehensive collection of appropriate axioms from which it is possible to extract the remaining thermodynamics. Such laws illustrate temperature equality as the zero law by Bekenstein [4] and Smarr [5], energy covservation as first law, entropy affiliations as second law or the third thermodynamic law for the absence of temperature.The strengthening of such laws took place after new invention of Hawking radiation.

The fruitful relation between the equation of the Einstein field and the first law of black hole thermodynamics be firstly explore by Jacobson in 1975 [6] and then Padmanab- han [7] found a formula to explain the thermodynamics for horizons in space-time.. He proved that in the context of first law of thermodynamics, we could rewrite the Einstein equations as:

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$$TdS = dE + PdV. \quad (1)$$

After then there is a stream of work in that direction for different black hole geometries [8-16]and [32-37].

Foliation and First Law of Black Hole Thermodynamics

The term foliation, that means leaf, is from original Latin folia. In the seventeenth century, the idea of foliation and slicing emerged from the concept of differential equa- tions in which the trajectories of solution space could be considered even as foliation leaves. In the end of the nineteenth century, Poincare was able to construct methods for theoretical analysis of dynamical systems' global, qualitative characteristics in cir- cumstances where explicit methods of solution had collapsed. He finds that study of a geometry of space of the dynamical system's trajectories shows complex phenomena. A decomposition for M through sub manifolds is just the foliation of the n−dimensional manifold, M , that are all of same dimension, p. The sub manifolds are the foliating leaves. The co-dimensions q are defined as $q = n - p$ of the foliation. The foliation by hyper surfaces is considered as foliation of co-dimension 1. Reeb [17] with Ehresmann
[18] were the founders of the philosophy of foliation, and the former invented the word foliation in particular. Some physically appropriate solutions are unique and comprise black hole space-time Stephani [19] to Einstein field's equations. Such space-times have particular relevance due to horizons through their geometries. Foliates [20-31] are required for evaluation of dynamics of such types of geometries by null or space-like

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(a) (b)(c) Now C show the constant structure of 3 factors be positioned in groups G and ς (Coupling constant),In YM potentials A(a) are the SO(3) measurements collections.Here solution of EPYM black hole with negative cosmological constant Λ is:

$$ds^2 = -Y(r)dt^2 + \frac{dr^2}{Y(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (4)$$

with Y (r) are now defined by:

$$Y(r) = 1 - \frac{2M}{r} - \frac{\Delta r^2}{3}\frac{(2q^2)^{\gamma}}{+ 2(4\gamma - 3)r^{(4\gamma-2)}} \quad (5)$$

$d\Omega_2^2$ represents unit 2-sphere metric and charge q. We Consider the two cases in next subsections possessing black hole solutions at $\gamma = 1$ and $\gamma = 1$ respectively.

First Law of Black Hole Thermodynamics For a Black Hole in Power Yang-Mills Gravity with gamma = 1

Consider r =constant= k,we get induced metric as

$$ds^2 = -Y(k)dt^2 + k^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (6)$$

where

$$Y(k) = 1 - k\frac{2M}{=} \frac{\Delta k^2}{3} \frac{q^2}{+ k^2}. \quad (7)$$

We discuss the thermodynamic properties of a black hole in Power Yang-Mills Gravity at the outer horizon r+. The Yang-Mill spacetime is associated with temperature T ,entropy S and electric potential ϕ as

$$T = \frac{6Mr_+ - 2\Lambda r^4 - 6q^2}{12r^3\pi + } \quad (8)$$

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$S = A = \pi r2$ (9)

and $\frac{}{4}$ +

$\phi = \frac{\partial M}{\partial q}|_{r=r+} = \frac{q}{r}$. (10)

The mass M of the black hole is given by

$M = 2 - \frac{r}{2r} - \frac{\Lambda r3}{} + \frac{q2}{}$, (11)

The (0,0)-component of the field equation for the induced metric (6)can be written in the form $\frac{1}{}(-3k2 + 6Mk + \Lambda k4 - q2)$

$$G_{0,0} = \frac{}{k^3} \quad . \qquad (12)$$

Again the (0, 0) part of the field equation for induced metric by taking T00 = −P , where P refers to the source's pressure, after multiplying it by dk and treating k as a parameter gives:

$\frac{dk}{2} = k\,dk - \frac{M}{} + \frac{\Lambda k2dk}{6} - \frac{q2dk}{6k2} = 4\pi Pk\,dk.$ (13)

On the outer horizon, from Eq.(11), we have

$dM = \frac{dr+}{2} - \frac{\Lambda r2\,dr+}{3} + \frac{qdq}{r+} - \frac{q2dr+}{2r2}$ (14)

Replacing equation (14) in equation (13) and simplifying at outer horizon r+, we get:

$dM - \frac{6Mr+ - 2\Lambda r4}{12\pi r3} + \frac{}{6q} d(\pi r2) - \frac{q}{r} dq - \frac{q2}{3r2} dk = Pd(\frac{4}{3}\pi r3)$ (15)

Utilizing thermal quantities, expressed after equation (7), takes the form of first law:

$dE = Tds + \phi dq + PdV.$ (16)

Here the volume surrounded by horizon is $V = \frac{4}{3}\pi r3$ and $dE = dM - q\,dk.$

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Which is the desired first law of thermodynamics for a black hole in Power Yang-Mills gravity with $\gamma = 1$.

2.2 First Law of Black Hole Thermodynamics For a Black Hole in Power Yang-Mills Gravity with gamma = 1

$$Y(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} - \sqrt{2}\frac{q^2}{r^2} \quad (17)$$

Here M and q are respectively the mass and charge of a black-hole. Temperature $T = \frac{3M\Lambda r^3}{6\pi r^2}$, entropy $S = A = \pi r^2$ (where A is the horizon area) and electrical potential $\phi = \partial M|_{r=r_+} = \frac{r_+ dq}{2\sqrt{2}}$ are the thermal quantities connected to black hole on the extreme horizon r_+.

A foliation of the space time with r =constant= k hyper surfaces is again assumed, such that the horizon of the black hole belongs to a given hyper surface. The hyper surfaces' induced metric takes the equivalent form calculated using equation (3), by f now from equation (15).Consider (0, 0)-the part of Einstein tensor for the induced metric to search for the dynamics of the hyper surfaces.

$$G_{0,0} = \frac{1}{3}\frac{(-3r\sqrt{2} + 6M\sqrt{2} + \Lambda r^3\sqrt{2} + 3rq)}{r^3\sqrt{2}} \quad (18)$$

Again the (0, 0) part of the Einstein field equation (1) for induced metric by $T00 = -P$, since multiplying it by dk, treats k as a parameter and considers an arbitrary slight displacement dk, and gives:

$$2 - k\,dk - \frac{dk}{M} - \frac{\Lambda k^2}{6}\,dk - 2\sqrt{2}\frac{q^2}{2}\,dk = 4\pi P k\,dk \quad (19)$$

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Where P refers to the source's pressure.Even on the horizon m = r − Λr3 − q√r . —=
2 6 2 2
dr+ Λr2 dr+ r+dq qdr+
+
dM = —2— − 2√2 − 2√2 (20) −
2
Replacing equation (20) in equation (19) and simplifying at outer horizon r+, we re-

ceive:

3 M − Λr3 ±
2 r+ 4 3
dM − 6πr2 d(πr+) − 2√2 dq = Pd(3 πr+) (21) – –
In forms of thermal quantities described after equation (17), equation (21) takes the first law form as:
dM = Tds + PdV + ϕdq. (22)

Here the volume surrounded by horizon is V = 4 πr3 .
3 +

3 Conclusion

In this research, we followed an approach that uses foliation to extract the first law at the outer horizon of black Holes in Power Yang-Mills spacetime gravity with γ = 1 and γ = 1 respectively. Spacetimes are foliated, in such a theme that the outer horizon of the spacetime represents a particular leaf (hypersurface) of the foliation. Then working out the field's equation of the lowered dimensional induced metric of the hypersurfaces and showed that the field equations for the induced metric, at the outer horizon, can be expressed as the black hole thermodynamic's first law. Here the notable aspect is that one will have to fundamentally contract with 3−dimensional induced metric for an 4−dimensional spacetime,which is significant to simplify the calculations to derive such results of complex spacetime gravities. It will be stimulating to encompass this approach to the other black hole gravities in future.

Online ISSN 3007-3197
Print ISSN 3007-3189

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