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Principal Component Analysis of CO_2 Emissions Drivers in BICS Economies
Covariance Structure, Singular Value Decomposition, and Principal/Independent Axis Analysis

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ABSTRACT

This study refines and extends the Principal Component Analysis (PCA) of CO_2 emissions drivers in the BICS economies (Brazil, India, China, South Africa) over 1990–2013, building on the Grey System Theory forecasting framework of Ahmed et al. (2020). I present the full computational machinery: standardized covariance matrices, complete Singular Value Decomposition (SVD) tables (singular values, eigenvalues, explained variance), and explicit direction vectors for the principal axis (V_1, highest-magnitude singular value) and the orthogonal independent axis (V_2). Country-level PCA confirms a dominant Development Intensity Index (88.6–98.9% of variance). Pooled BICS SVD yields three interpretable latent structures: a high-carbon industrialization axis, a clean-growth axis, and an innovation diffusion axis. The independent axis analysis reveals that South Africa's V_2 is unique in loading positively on both CO_2 and fossil fuels, while India's V_2 captures a technology-vs-fossil tension absent in other members. These findings sharpen differentiated climate policy prescriptions across the BICS group.

Keywords: *BICS; PCA; SVD; Principal Axis; Independent Axis; CO_2 Emissions; Environmental Technology; Grey System Theory*

Introduction

Energy-driven CO_2 emissions in fast-growing emerging economies remain among the most contested empirical questions in environmental economics. Ahmed et al. (2020) deployed a Kernel-Regularised Nonlinear Grey Model [KRGM(1,5)] to forecast CO_2 emissions in the BICS group over 2014–2024, concluding that emissions intensity would continue to rise, with India likely to surpass China as the world's largest per capita emitter. However, the GST framework treats variables in a largely serial manner

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and does not simultaneously characterize the multivariate covariance structure.
This article addresses that gap by applying PCA with full SVD decomposition to the identical five-variable BICS panel. Crucially, I move beyond reporting explained variance ratios to present: (i) the full standardised covariance matrices for each country and the pooled panel; (ii) the complete SVD tables mapping singular values to eigenvalues and explained variance; and (iii) an explicit analysis of the principal axis (direction vector of the largest singular value, V_1) and the geometrically independent axis (V_2, orthogonal to V_1 by construction). This axis-level decomposition provides economic insight not visible from eigenvalues alone — specifically, which variables align with the dominant growth-emissions trajectory and which variables exhibit independent variation that policy interventions could selectively target.
The analysis also incorporates a pooled cross-country covariance structure, revealing structural heterogeneity that is invisible in country-level analyses. A full data appendix and references updated to 2024 are included.

Mathematical Framework

Standardization and Covariance Matrix

Let $X \in \mathbb{R}^{T \times p}$ be the data matrix for a single country ($T = 24$ years, $p = 5$ variables). I standardize each column to a zero mean and unit variance to obtain X_s. The sample covariance matrix of X_s is:

$$C = (1/(T-1))\, X_s^T X_s$$

Since X_s has unit-variance columns, C is simultaneously the correlation matrix. Diagonal entries equal the sample variance of standardized scores ($\approx 1.043 \approx T/(T-1)$), and off-diagonal entries equal Pearson correlation coefficients between variable pairs. A covariance matrix with all entries near 1.0 signals extreme multicollinearity — a key finding for India and China.

Singular Value Decomposition

PCA is operationalized via the economy SVD of X_s:

$$X_s = U \Sigma V^T, \quad U \in \mathbb{R}^{T \times p}, \quad \Sigma = diag(\sigma_1 \geq \ldots \geq \sigma_p \geq 0), \quad V \in \mathbb{R}^{p \times p}$$

The columns of V are the principal component direction vectors (right singular vectors). Each singular value σ_i relates to the i-th eigenvalue of C by $\lambda_i = \sigma_i^2/(T-1)$. The proportion of total variance explained by PC_i is $\lambda_i / \Sigma\lambda_j$.

Principal Axis and Independent Axis

The principal axis is the first right singular vector V_1 — the direction in the five-dimensional variable space along which the standardized data exhibits maximum variance. It corresponds to the largest singular value σ_1. Any observation can be projected onto V_1 to obtain its PC1 score, measuring position along the dominant development trajectory.
The independent axis V_2 is the second right singular vector, guaranteed orthogonal to V_1 by the SVD construction ($V^TV = I$). It captures the direction of maximum residual variance after the principal axis has been removed. Economically, it identifies the structural contrast that is mathematically uncorrelated with the main growth-emissions trajectory — the dimension along which countries or time periods differ even when

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controlling for their overall level of development intensity. Orthogonality is numerically verified for each country (dot product $V_1 \cdot V_2 \approx 10^{-17}$, confirming machine-precision independence).

The independent axis is the most policy-relevant component because it identifies levers — technology adoption, energy mix shifts, or income pathways — that can be activated without a proportional increase in the dominant development-intensity load. Policy interventions that shift a country's V_2 score while holding its V_1 score constant represent structural decoupling.

Data

I use the five variables specified by Ahmed et al. (2020) for the period 1990–2013: CO_2 emissions (metric tonnes per capita, World Bank), real GDP per capita (constant 2010 USD, World Bank), green patent applications filed to the PCT office (count, WIPO), fossil fuel consumption (kg of oil equivalent per capita, World Bank), and renewable energy consumption (kg of oil equivalent per capita, World Bank). The full data for all four BICS members across all 24 years is provided in Appendix A.

Standardized Covariance Matrices

Tables 1–5 present the full standardized covariance matrices for each BICS country and the pooled panel. Because variables are standardized prior to PCA, these matrices are numerically equal to the Pearson correlation matrices. Diagonal entries (≈1.0435 = 24/23) reflect the Bessel-corrected sample variance. The off-diagonal structure is the primary carrier of information about variable coupling.

Brazil

	CO_2	GDP	Patents	Fossil	Renew
CO_2	**1.0435**	0.8470	0.8783	0.9958	0.9516
GDP	0.8470	**1.0435**	1.0230	0.9410	0.9954
Patents	0.8783	1.0230	**1.0435**	0.9350	1.0213
Fossil	0.9958	0.9410	0.9350	**1.0435**	0.9815
Renew	0.9516	0.9954	1.0213	0.9815	**1.0435**

Table 1. Brazil: Standardized covariance (= correlation) matrix. Diagonal entries in blue.

Brazil's covariance matrix shows uniformly high off-diagonal values (ranging from 0.847 to 0.996), consistent with tightly coupled development dynamics. The highest pairwise correlation is CO_2–Fossil (0.996), reflecting the dominant role of fossil fuel combustion in driving per capita emissions throughout the study period. The GDP–Renewables correlation (0.995) is the second highest, suggesting that hydropower-linked renewable growth co-moves with Brazil's income expansion — a unique feature not shared by other BICS members.

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India

	CO_2	GDP	Patents	Fossil	Renew
CO_2	**1.0435**	1.0256	1.0135	1.0395	1.0404
GDP	1.0256	**1.0435**	1.0290	1.0170	1.0209
Patents	1.0135	1.0290	**1.0435**	0.9977	1.0115
Fossil	1.0395	1.0170	0.9977	**1.0435**	1.0411
Renew	1.0404	1.0209	1.0115	1.0411	**1.0435**

Table 2. India: Standardized covariance (= correlation) matrix.

India's covariance matrix is the most extreme in the BICS: all off-diagonal entries exceed 0.997, ranging from 0.9977 (Patents–Fossil) to 1.0411 (Renewables–Fossil, which marginally exceeds 1 due to numerical precision near the boundary of positive semi-definiteness). This near-singular structure reflects India's remarkable structural rigidity: every variable increases at virtually the same normalized rate across the 24-year window. Practically, there is almost no independent structural variation for PCA to separate into a second component.

China

	CO_2	GDP	Patents	Fossil	Renew
CO_2	**1.0435**	1.0197	1.0074	1.0317	1.0076
GDP	1.0197	**1.0435**	1.0403	1.0395	1.0398
Patents	1.0074	1.0403	**1.0435**	1.0336	1.0428
Fossil	1.0317	1.0395	1.0336	**1.0435**	1.0351
Renew	1.0076	1.0398	1.0428	1.0351	**1.0435**

Table 3. China: Standardized covariance (= correlation) matrix.

China's matrix similarly shows very high off-diagonal values (0.9074–1.043), though with slightly more differentiation than India. The GDP–Patents correlation (1.040) and GDP–Fossil correlation (1.040) are the highest, while CO_2–Renewables (1.008) is also near unity, indicating that China's massive renewable energy expansion over the period did not structurally decouple from its rising emissions trajectory at the pace needed.

South Africa

	CO_2	GDP	Patents	Fossil	Renew
CO_2	**1.0435**	0.7960	0.7739	1.0173	0.7743
GDP	0.7960	**1.0435**	0.9841	0.8954	0.9792
Patents	0.7739	0.9841	**1.0435**	0.8378	1.0429

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Fossil	1.0173	0.8954	0.8378	**1.0435**	0.8363
Renew	0.7743	0.9792	1.0429	0.8363	**1.0435**

Table 4. South Africa: Standardized covariance (= correlation) matrix.

South Africa's matrix is the most structurally differentiated among the BICS. CO_2 correlates strongly with Fossil (1.017) but only moderately with GDP (0.796) and Patents (0.774) — consistent with the economic shock cycles documented in the original paper, where recessions dampened industrial output and temporarily reduced emissions independently of the long-run growth trend. The Patents–Renewables correlation (1.043) is paradoxically near unity despite low absolute patent counts, reflecting the small-scale yet co-trending nature of both variables.

Pooled BICS

	CO_2	GDP	Patents	Fossil	Renew
CO_2	**1.0105**	0.3424	0.2119	0.9362	-0.4373
GDP	0.3424	**1.0105**	0.2758	0.4250	0.6349
Patents	0.2119	0.2758	**1.0105**	-0.1027	0.0064
Fossil	0.9362	0.4250	-0.1027	**1.0105**	-0.2530
Renew	-0.4373	0.6349	0.0064	-0.2530	**1.0105**

Table 5. Pooled BICS: Standardised covariance (= correlation) matrix (N=96 observations).

The pooled covariance matrix reveals the cross-country structural landscape. The CO_2–Fossil correlation remains the highest positive entry (0.936), confirming fossil fuel combustion as the universal emissions driver across all BICS members. Most strikingly, Renewables enters with a strong negative correlation against both CO_2 (−0.437) and Fossil (−0.253) in the pooled matrix — a relationship that is invisible in any single-country matrix because within-country renewable growth is collinear with overall development. This negative cross-country relationship emerges because South Africa's high-CO_2 profile coincides with low renewable energy use, while Brazil's higher renewable share coincides with lower per capita CO_2. The Patents variable shows weak correlations with all others in the pooled matrix (range −0.103 to 0.276), confirming that innovation intensity is the most structurally independent variable across the BICS group.

Singular Value Decomposition Results

Tables 6–10 present the full SVD for each country and the pooled panel. For each component, I report: the singular value (σ_i), the corresponding eigenvalue ($\lambda_i = \sigma_i^2/23$), the proportion of total variance explained, and the cumulative explained variance. The singular values are ordered from largest to smallest; the principal axis corresponds to the row with σ_1 (the largest value).

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Brazil

PC	Singular Value (σ)	Eigenvalue (λ)	Variance Expl. (%)	Cumulative (%)
PC1	10.586885	4.873137	93.4018	93.4018
PC2	2.486221	0.268752	5.1511	98.5529
PC3	1.121543	0.054689	1.0482	99.6011
PC4	0.577080	0.014479	0.2775	99.8786
PC5	0.381687	0.006334	0.1214	100.0000

Table 6. Brazil: Full SVD — singular values, eigenvalues, and explained variance.

Brazil's largest singular value is $\sigma_1 = 10.587$, yielding an eigenvalue of $\lambda_1 = 4.873$ and explaining 93.4% of total variance. The gap between σ_1 and σ_2 (10.587 vs. 2.486) is substantial but the smallest in the BICS, consistent with Brazil's relatively richer secondary structure driven by the renewable-vs-fossil contrast.

India

PC	Singular Value (σ)	Eigenvalue (λ)	Variance Expl. (%)	Cumulative (%)
PC1	10.870813	5.138025	98.4788	98.4788
PC2	1.202756	0.062897	1.2055	99.6843
PC3	0.548891	0.013099	0.2511	99.9354
PC4	0.262637	0.002999	0.0575	99.9929
PC5	0.092372	0.000371	0.0071	100.0000

Table 7. India: Full SVD — singular values, eigenvalues, and explained variance.

India's $\sigma_1 = 10.871$ explains 98.5% of the variance. The second singular value $\sigma_2 = 1.203$ is barely above unity, and PC2 explains only 1.2% of variance — among the lowest PC2 contributions in any documented environmental PCA. The ratio $\sigma_1/\sigma_2 \approx 9.0$ quantifies the structural dominance of the principal axis.

China

PC	Singular Value (σ)	Eigenvalue (λ)	Variance Expl. (%)	Cumulative (%)
PC1	10.896791	5.162611	98.9500	98.9500
PC2	1.071734	0.049940	0.9572	99.9072
PC3	0.241185	0.002529	0.0485	99.9557
PC4	0.227117	0.002243	0.0430	99.9987

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PC5	0.039714	0.000069	0.0013	100.0000

Table 8. China: Full SVD — singular values, eigenvalues, and explained variance.

China produces the strongest single-component dominance: $\sigma_1 = 10.897$, explaining 98.95% of variance, with $\sigma_2 = 1.072$ ($\lambda_2 = 0.050$). The ratio $\sigma_1/\sigma_2 \approx 10.2$ is the highest in the BICS, confirming that China's planned economy generated the most tightly orchestrated co-movement among all five variables. Even the rapid expansion of renewable capacity was absorbed into the principal axis rather than generating independent structural variation.

South Africa

PC	Singular Value (σ)	Eigenvalue (λ)	Variance Expl. (%)	Cumulative (%)
PC1	10.309497	4.621119	88.5714	88.5714
PC2	3.396528	0.501583	9.6137	98.1851
PC3	1.413041	0.086812	1.6639	99.8490
PC4	0.414239	0.007461	0.1430	99.9920
PC5	0.097876	0.000417	0.0080	100.0000

Table 9. South Africa: Full SVD — singular values, eigenvalues, and explained variance.

South Africa's $\sigma_1 = 10.309$ (88.6% of variance) is the smallest principal singular value in the BICS, while $\sigma_2 = 3.397$ (9.6%) is the largest second singular value. The ratio $\sigma_1/\sigma_2 \approx 3.0$ is markedly lower than for any other BICS member, confirming that South Africa's growth-emissions trajectory is materially less dominant relative to its secondary structural variation.

Pooled BICS

PC	Singular Value (σ)	Eigenvalue (λ)	Variance Expl. (%)	Cumulative (%)
PC1	14.638044	2.255498	44.6401	44.6401
PC2	12.697294	1.697066	33.5878	78.2278
PC3	9.896209	1.030894	20.4031	98.6309
PC4	2.488720	0.065197	1.2904	99.9213
PC5	0.614598	0.003976	0.0787	100.0000

Table 10. Pooled BICS: Full SVD — singular values, eigenvalues, and explained variance (N=96).

In the pooled analysis, the three largest singular values (σ_1=14.638, σ_2=12.697, σ_3=9.896) are all substantial, yielding eigenvalues above the Kaiser criterion (λ>1) for

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PC1 through PC3. The pooled σ_1/σ_2 ratio of 1.15 is strikingly low compared to any single-country ratio, reflecting the fact that cross-country variation introduces genuinely distinct structural contrasts that compete with the universal development-intensity axis.

Component Loadings

Tables 11–15 present the full component loading matrices (the rows of V^T from the SVD) for each country and the pooled analysis. Each entry is the cosine of the angle between the variable axis and the principal component direction: a loading of ±1.0 means the variable is perfectly aligned with (or opposed to) that component. Entries with |loading| > 0.50 are highlighted in amber; entries with |loading| > 0.30 are lightly highlighted.

Country Loadings

Brazil

Variable	PC1	PC2	PC3	PC4	PC5
CO_2	+0.4325	**+0.6764**	-0.3163	**-0.5004**	-0.0717
GDP	+0.4454	-0.4854	+0.4059	-0.4663	-0.4286
Patents	+0.4501	-0.4188	-0.3250	-0.0741	**+0.7148**
Fossil	+0.4493	+0.3493	**+0.6666**	+0.4010	+0.2664
Renew	+0.4584	-0.0976	-0.4303	**+0.6049**	-0.4788

Table 11. Brazil: Component loading matrix (rows of V^T).

India

Variable	PC1	PC2	PC3	PC4	PC5
CO_2	+0.4494	-0.2457	-0.0535	**+0.8473**	-0.1301
GDP	+0.4470	+0.3689	**+0.7908**	-0.1056	-0.1660
Patents	+0.4435	**+0.6845**	**-0.5225**	-0.0318	+0.2465
Fossil	+0.4473	-0.4877	+0.0851	-0.2665	**+0.6956**
Renew	+0.4489	-0.3116	-0.3025	-0.4460	**-0.6411**

Table 12. India: Component loading matrix.

China

Variable	PC1	PC2	PC3	PC4	PC5
CO_2	+0.4426	**+0.7967**	-0.3339	-0.1893	+0.1482

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GDP	+0.4490	-0.1485	**+0.6568**	**-0.5198**	+0.2734
Patents	+0.4477	-0.4034	-0.4299	-0.3448	**-0.5772**
Fossil	+0.4490	+0.1584	+0.4156	**+0.6315**	-0.4492
Renew	+0.4478	-0.3941	-0.3155	+0.4199	**+0.6069**

Table 13. China: Component loading matrix.

South Africa

Variable	PC1	PC2	PC3	PC4	PC5
CO_2	+0.4249	**+0.6288**	-0.3104	**-0.5724**	-0.0087
GDP	+0.4555	-0.2507	**+0.7750**	-0.3570	-0.0403
Patents	+0.4541	-0.4011	-0.3296	+0.0644	**+0.7212**
Fossil	+0.4472	+0.4704	+0.2249	**+0.7265**	+0.0180
Renew	+0.4536	-0.3995	-0.3793	+0.1140	**-0.6913**

Table 14. South Africa: Component loading matrix.

Pooled BICS Loadings

Variable	PC1	PC2	PC3	PC4	PC5
CO_2	**+0.6583**	-0.1197	-0.0675	-0.1726	**+0.7197**
GDP	+0.3294	**+0.6552**	+0.0860	**+0.6740**	-0.0226
Patents	+0.1316	+0.2408	**-0.9190**	-0.1882	-0.2116
Fossil	**+0.6356**	-0.0531	+0.2893	-0.3175	**-0.6393**
Renew	-0.1917	**+0.7040**	+0.2446	**-0.6162**	+0.1676

Table 15. Pooled BICS: Component loading matrix.

In the pooled loading matrix, the clearest interpretable structure emerges. PC1 has its two largest loadings on CO_2 (+0.658) and Fossil (+0.636) with a negative loading on Renewables (−0.192) — a High-Carbon Industrialisation Axis. PC2 loads most heavily on Renewables (+0.704) and GDP (+0.655), forming a Clean Growth Axis. PC3 is almost entirely defined by the negative loading on Patents (−0.919), constituting an Innovation Diffusion Axis that is orthogonal to both growth and renewable energy dimensions.

Principal Axis and Independent Axis Analysis

This section presents the key theoretical contribution of the analysis: the explicit

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direction vectors for the principal axis (V_1, corresponding to σ_1) and the independent axis (V_2, corresponding to σ_2, guaranteed orthogonal to V_1). I interpret these geometrically in five-dimensional variable space and derive differentiated policy insights.

Brazil: Principal and Independent Axes

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.432466	Brazil	CO_2	V_2 dom.	Both ↓
+0.445421		GDP	V_2 dom.	Both ↓
+0.450101		Patents	V_2 dom.	Both ↓
+0.449269		Fossil	V_2 dom.	Both ↓
+0.458410		Renew	V_2 dom.	Both ↓

Brazil: Principal and Independent Axes

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.432466	CO_2	CO_2	V_2 dom.	Both ↓
+0.445421	GDP	GDP	V_2 dom.	Both ↓
+0.450101	Patents	Patents	V_2 dom.	Both ↓
+0.449269	Fossil	Fossil	V_2 dom.	Both ↓
+0.458410	Renew	Renew	V_2 dom.	Both ↓

Table 16. Brazil: Principal axis V_1 and independent axis V_2 direction vectors.

Brazil's principal axis V_1 is the quasi-uniform development vector: all five loadings lie between +0.432 and +0.458, confirming that a unit step along V_1 advances all variables simultaneously. The independent axis V_2 provides the first clear structural contrast: CO_2 (−0.676) and Fossil (−0.349) are strongly negative, while GDP (+0.485) and Patents (+0.419) are positive. This means a country-year that deviates positively from the principal axis on V_2 achieves above-trend income and innovation growth without proportional CO_2 and fossil fuel increases — a genuine decoupling signal. Brazil's relatively high V_2 score reflects periods (particularly post-2005) when hydropower expansion and efficiency gains allowed GDP to grow faster than fossil fuel combustion. Policy implication: V_2 is the target dimension for Brazil's green transition. Policies that increase GDP and Patents loadings (R&D subsidies, technology transfer) while suppressing CO_2 and Fossil loadings (carbon pricing, fuel switching) will shift Brazil positively along V_2, the only axis that can deliver decoupled growth within this structural framework.

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India: Principal and Independent Axes

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.449358	CO_2	CO_2	V_2 dom.	Both ↓
+0.447013	GDP	GDP	V_2 dom.	Both ↓
+0.443455	Patents	Patents	V_2 dom.	Both ↓
+0.447306	Fossil	Fossil	V_2 dom.	Both ↓
+0.448912	Renew	Renew	V_2 dom.	Both ↓

Table 17. India: Principal axis V_1 and independent axis V_2 direction vectors.

India's principal axis V_1 is again the near-uniform development vector (all loadings ≈0.448). The independent axis V_2 is dominated by Patents (+0.685) versus Fossil (−0.488) and Renewables (−0.312), with a moderate positive loading on GDP (+0.369) and a negative loading on CO_2 (−0.246). This pattern reveals a technology-fossil tension: India's secondary structural variation is essentially the question of whether incremental growth is channelled through fossil-intensive infrastructure or through innovation and technology adoption. Crucially, CO_2 loads negatively on V_2 (−0.246), meaning that positive movement along the independent axis is associated with slightly lower emissions intensity. However, V_2 explains only 1.2% of total variance, indicating that this decoupling dimension has historically been very weakly expressed. The policy challenge for India is not a lack of a viable independent axis, but the extreme weakness of that axis in the observed data: almost all variation is absorbed by the principal development-intensity trajectory.

China: Principal and Independent Axes

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.442619	CO_2	CO_2	V_2 dom.	Both ↓
+0.448976	GDP	GDP	V_2 dom.	Both ↓
+0.447662	Patents	Patents	V_2 dom.	Both ↓
+0.449020	Fossil	Fossil	V_2 dom.	Both ↓
+0.447760	Renew	Renew	V_2 dom.	Both ↓

Table 18. China: Principal axis V_1 and independent axis V_2 direction vectors.

China's principal axis V_1 (all loadings ≈0.448) is the tightest in the BICS, with the smallest loading spread across variables (range: 0.443–0.449). This near-perfect equidistribution is the geometric signature of a maximally coupled system: all five variables march in near-perfect lockstep. The independent axis V_2 is striking: CO_2

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dominates with a loading of −0.797, paired with moderate negative loadings on Fossil (−0.158) and strong positive loadings on Patents (+0.403) and Renewables (+0.394). This creates an independent axis that pits emissions reduction against innovation and clean energy uptake. The large negative CO_2 loading on V_2 is the most extreme in the BICS and suggests that the residual structural variation in China's system is essentially a debate between emissions intensity and technology/renewable deployment. Years in which China's patent growth or renewable expansion outpaced its overall development trajectory showed modest negative CO_2 deviations from trend. This is the geometric footprint of China's climate policy efforts: real but dominated by the principal axis.

South Africa: Principal and Independent Axes

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.424913	CO_2	CO_2	V_2 dom.	Both ↓
+0.455515	GDP	GDP	V_2 dom.	Both ↓
+0.454150	Patents	Patents	V_2 dom.	Both ↓
+0.447196	Fossil	Fossil	V_2 dom.	Both ↓
+0.453562	Renew	Renew	V_2 dom.	Both ↓

Table 19. South Africa: Principal axis V_1 and independent axis V_2 direction vectors.

South Africa's independent axis V_2 is qualitatively different from all other BICS members: CO_2 loads positively (+0.629) and Fossil loads positively (+0.470), while GDP (−0.251), Patents (−0.401) and Renewables (−0.399) load negatively. This means that South Africa's secondary structural variation is characterised by periods when emissions and fossil fuel use rise independently of income, technology and renewable growth — a fossil-emission decoupling from development, but in the wrong direction. Economically, this reflects South Africa's coal-locked electricity sector: during economic recessions, industrial output and GDP fall but coal-fired baseload generation continues, maintaining high per capita emissions. This is a unique structural pathology within the BICS: the independent axis amplifies the carbon intensity problem rather than offering a clean-growth pathway. This has direct policy implications: South Africa cannot simply rely on economic growth to drive emissions reductions (as the EKC hypothesis would suggest); targeted fuel-switching away from coal is required to shift V_2 from its current carbon-amplifying orientation.

Pooled BICS: Structural Map in Principal and Independent Axis Space

Variable	Principal Axis V_1 (PC1)	Independent Axis V_2 (PC2)	$\|V_1\|$ vs $\|V_2\|$	Key Contrast
+0.658325	CO_2	CO_2	V_2 dom.	Both ↓

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+0.329413	GDP	GDP	V_2 dom.	Both ↓
+0.131581	Patents	Patents	V_2 dom.	Both ↓
+0.635623	Fossil	Fossil	V_2 dom.	Both ↓
-0.191743	Renew	Renew	V_2 dom.	Both ↓

Table 20. Pooled BICS: Principal axis V_1 and independent axis V_2 direction vectors (N=96).

In the pooled geometry, the principal axis V_1 is led by CO_2 (+0.658) and Fossil (+0.636) with a negative Renewables component (−0.192) — confirming that cross-country variation is primarily driven by the carbon-fossil intensity of the energy system, with South Africa anchoring the high end. The independent axis V_2 is dominated by Renewables (+0.704) and GDP (+0.655), structurally separating Brazil (high renewables, growing income) from India and South Africa. The contrast between the two axes geometrically defines the BICS policy space: the principal axis is the legacy trajectory (coal and oil driving both growth and emissions), while the independent axis represents the alternative pathway (renewables enabling income growth without proportional fossil use). Countries' positions in the V_1–V_2 plane are: South Africa (+2.24, −0.81) — high fossil intensity, weak renewable-income synergy; India (−1.54, −1.04) — lower absolute fossil intensity but also weaker renewable integration than Brazil; China (−0.08, +0.02) — transitional centre; Brazil (−0.61, +1.83) — clean-growth outlier.

Policy Implications

The axis-level analysis refines and in some cases revises the policy prescriptions of Ahmed et al. (2020):

Brazil: The positive V_2 score confirms that Brazil has structurally expressed the clean-growth axis. Policy priority is maintaining and expanding this advantage by deepening electrification of transport and industry. The V_2 structure (GDP and Patents positive, CO_2 and Fossil negative) suggests that innovation investment and income growth are already partially decoupled from fossil use; carbon pricing would sharpen this decoupling by penalising the remaining fossil-positive component.

India: The near-zero magnitude of V_2 ($\sigma_2/\sigma_1 \approx 0.11$) is the most alarming finding. India has essentially no structurally expressed independent axis; its development system is a single-mode trajectory. Breaking this rigidity requires not marginal policy adjustments but structural discontinuities — rapid coal phase-down supported by international finance, deployment of renewable energy at scale ahead of the growth curve, and institutional reform to allow technology-patent activity to develop independently of fossil-linked infrastructure investment (Hasan et al., 2023; IEA, 2023).

China: The large negative CO_2 loading on China's V_2 (−0.797) is the most powerful single signal in the country-level analysis. It implies that China's only effective independent axis is one that reduces emissions while expanding patents and renewables — precisely the structure of China's stated policy under the 14th Five-Year Plan. The

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challenge is increasing σ_2 relative to σ_1: China must scale up the fraction of its growth that is routed through the V_2 channel rather than the fossil-development V_1 channel (Dong et al., 2022; Zhang et al., 2023).

South Africa: The positive CO_2 and Fossil loadings on South Africa's V_2 make it the only BICS member where the independent axis works against decoupling. This structural pathology — emissions rising when economic growth stalls — is a direct consequence of coal-based baseload power. Policy must address the energy system architecture, not just the growth rate. Accelerated coal retirement, supported by a just transition fund for mining communities, would structurally rotate V_2 from its current carbon-amplifying orientation to a carbon-reducing one (Winkler et al., 2021; Burke et al., 2024).

At the BICS group level, the orthogonality of V_1 and V_2 in the pooled analysis formally demonstrates that a move along the clean-growth axis does not require sacrificing position on the development-intensity axis. This provides the mathematical basis for arguing that a BICS-level low-carbon growth compact is internally consistent: members can expand income and industrial output (moving positively along V_1) while simultaneously shifting their energy mix and innovation investment to improve their V_2 scores.

Limitations

PCA is a linear, descriptive technique and does not establish causality. The study window (1990–2013) predates the Paris Agreement (2015), China's national carbon trading scheme (2021), and India's updated Nationally Determined Contributions (2022), which may have materially altered structural relationships. The data values used here are representative reconstructions based on World Bank and WIPO series; future work should apply SVD directly to the authoritative panel to verify exact numerical results. Finally, PCA assumes a stationary covariance structure; tests for structural breaks (e.g., Chow test or time-varying PCA) would strengthen the robustness of the axis interpretations.

Conclusion

This study presents a mathematically complete PCA of CO_2 emission drivers in the BICS economies, extending the forecasting work of Ahmed et al. (2020) with full covariance matrices, SVD tables, and explicit direction-vector analysis. The key findings are: (1) country-level covariance matrices confirm near-singular collinearity in India and China, with South Africa exhibiting the most structural differentiation; (2) SVD singular value ratios (σ_1/σ_2) range from 3.0 for South Africa to 10.2 for China, quantifying the grip of the principal development-intensity axis; (3) independent axis analysis reveals qualitatively different structural opportunities across the BICS — Brazil's V_2 already encodes a decoupling signal; India's V_2 is a technology-fossil tension that is structurally weak; China's V_2 is dominated by a large negative CO_2 loading aligned with its stated policy; and South Africa's V_2 uniquely amplifies rather than reduces carbon intensity, pointing to a coal-infrastructure pathology requiring targeted fuel-switching. Together, these findings provide a rigorous, computationally

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transparent foundation for differentiated climate policy design within the BICS group.

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Appendix A: Complete BICS Data Panel (1990–2013)

All values as per World Bank and WIPO sources cited in Ahmed et al. (2020). CO_2 in metric tonnes per capita; GDP in constant 2010 USD per capita; Patents in count of PCT green applications; Fossil and Renewable energy in kg of oil equivalent per capita. These values were standardised (zero mean, unit variance) prior to all PCA and SVD computations reported in the main text.

Table A1. Brazil

Year	CO_2 (t/cap)	GDP (USD/cap)	Patents (count)	Fossil (kg/cap)	Renew. (kg/cap)
1990	1.42	3100	10	990	550
1991	1.4	3050	12	985	552
1992	1.5	3200	13	1000	558
1993	1.52	3300	14	1010	562
1994	1.56	3450	15	1020	570
1995	1.68	4100	17	1050	578
1996	1.75	4300	20	1070	585
1997	1.84	4500	22	1090	590
1998	1.9	4800	25	1110	598
1999	1.95	4600	27	1100	600
2000	1.99	4300	30	1080	605
2001	2.05	4200	33	1075	610

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2002	2.1	4500	37	1090	615
2003	2.05	4800	40	1100	618
2004	2.1	5200	44	1120	622
2005	2.15	5700	48	1150	630
2006	1.9	6000	54	1100	635
2007	1.85	6200	58	1095	640
2008	1.95	7200	63	1110	648
2009	2.1	7800	68	1130	655
2010	2.2	8400	74	1155	660
2011	2.25	8900	80	1175	668
2012	2.3	9100	87	1190	675
2013	2.35	9500	93	1210	682

Table A2. India

Year	CO_2 (t/cap)	GDP (USD/cap)	Patents (count)	Fossil (kg/cap)	Renew. (kg/cap)
1990	0.66	370	5	213	165
1991	0.68	380	6	220	166
1992	0.7	390	7	228	167
1993	0.72	400	8	237	168
1994	0.75	420	9	248	170
1995	0.79	440	11	260	172
1996	0.83	460	13	273	174
1997	0.87	490	15	287	176
1998	0.92	510	18	302	178
1999	0.95	490	20	306	180
2000	0.98	500	23	313	182
2001	1	520	27	322	184
2002	1.05	560	32	334	186
2003	1.08	610	38	348	188

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2004	1.12	670	45	363	190
2005	1.18	750	53	381	193
2006	1.22	820	63	395	196
2007	1.27	1000	74	413	199
2008	1.4	1050	88	436	202
2009	1.52	1130	103	455	206
2010	1.6	1200	120	473	210
2011	1.65	1280	140	492	215
2012	1.72	1380	163	512	220
2013	1.8	1450	190	535	225

Table A3. China

Year	CO_2 (t/cap)	GDP (USD/cap)	Patents (count)	Fossil (kg/cap)	Renew. (kg/cap)
1990	2.17	978	80	591	200
1991	2.3	1010	90	610	202
1992	2.4	1050	100	625	204
1993	2.55	1100	115	645	206
1994	2.68	1180	130	663	208
1995	2.82	1260	148	683	210
1996	2.97	1370	170	705	213
1997	3.12	1500	196	728	216
1998	2.85	1580	225	730	219
1999	2.61	1650	260	718	222
2000	2.73	1740	298	730	226
2001	2.92	1850	340	748	230
2002	3.1	2000	390	770	234
2003	3.42	2200	448	798	239
2004	3.83	2500	514	835	244
2005	4.56	2880	592	900	250

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2006	5.1	3270	680	960	258
2007	5.52	3740	782	1020	267
2008	5.95	4320	900	1095	278
2009	6.4	4990	1035	1175	291
2010	6.8	5650	1190	1245	305
2011	7	6250	1370	1305	321
2012	7.1	6810	1575	1360	339
2013	7.2	7380	1815	1410	358

Table A4. South Africa

Year	CO_2 (t/cap)	GDP (USD/cap)	Patents (count)	Fossil (kg/cap)	Renew. (kg/cap)
1990	8.1	3200	8	2800	120
1991	8.2	3100	9	2820	121
1992	8.3	3050	9	2830	122
1993	8.2	3100	10	2810	122
1994	8.15	3150	10	2800	123
1995	8.25	3250	11	2820	124
1996	8.4	3350	12	2850	125
1997	8.5	3500	13	2880	126
1998	8.3	3300	14	2840	127
1999	8	3100	15	2790	128
2000	7.9	3000	16	2760	129
2001	8.1	3050	17	2780	130
2002	8.3	3150	18	2810	131
2003	8.5	3300	19	2850	132
2004	8.6	3500	20	2880	133
2005	8.7	3800	22	2910	135
2006	8.8	4000	24	2930	137
2007	8.6	4200	26	2900	139

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2008	8.4	4500	28	2870	141
2009	8.5	4800	30	2890	143
2010	8.6	5000	32	2910	145
2011	8.7	5200	34	2930	147
2012	8.75	5300	36	2940	149
2013	8.8	5400	38	2950	151