

Effect of Fluid Inertia on the Liquid Flow Phenomena through a Two-Step Curved Converging Channel

Abdul Hameed Wahocho*

Vice Principal Cadet College Khairpur, Sindh, Pakistan. Corresponding Author

Email: abdul116778@gmail.com

Abdul Sattar Chang

Research Scholar, Junior Elementary School Teacher (JEST), Education & Literacy

Department, Government of Sindh. sattarchang7@gmail.com

Hissam Uddin Shaikh

Chairman, Mathematics Department, Shah Abdul Latif University, Khairpur, Sindh,

Pakistan. hissam.shaikh@salu.edu.pk

Abdul Sami Junejo

M.Phil research scholar, Department of Mathematics, Shah Abdul Latif University,

Khairpur Mir's, Sindh. sami.junejo778@gmail.com

ABSTRACT

Finite element modeling is perceived with combine other approximate schemes, such as least squares, for the investigation of the flow features. The flow features are visualized via the two conservation laws (mass and momentum), and these laws are adopted for the steady state phenomena of numerical solution in the Cartesian coordinate form. The equations are derived for Newtonian fluids that behave only linearly and are visualized through the closed channel that is known as the curved double step channel. This channel relates to civil and hydraulic engineering to design the spillways and observe the hydraulic jumps. All predicted results are computed through the computational package COMSOL Multi Physics and this package is most applicable in the CFD discipline and provide unique plate form to understand mathematical theories of equations, solver settings imposed different numerical schemes that based upon the FEM discretization and many more post processing results focused on the fluid features, heat transfers and mass moved towards the closed and open bodies like channels, ducts, tubes, etc.

Keywords: Fluid Inertia, Liquid Flow Phenomena, Curved Converging Channel

Introduction

The concept of a detailed study of flow phenomena through a double-step curved backward channel is discussed. The flow structure is comprised of the steady state solution of the Navier–Stokes equations, which are a form of non–linear 2nd order partial differential equations. The computational approaches of the PDE’s are based upon the incompressible and Newtonian fluids that founded the field of modern technology-based computational fluid dynamics (CFD).

The CFD consequently grows day by day due to the enhancement of the latest computational packages of CFD, such as Matlab, Mathematica, ANSYS Gambit and fluent and the most popular COMSOL MultiPhysics. Also the modelling and simulation through CFD software has been observed as a hi–tec device to handle the low–tec and hi–tec real life mathematical problems such as Wang et al. (2014); Tinney and Ukeiley (2009); jay raj et al. (2013); and Mustafa et al. (2011) contributed the research work in the field of CFD through different tools of CFD’s. After studying the huge literature, the problem is chosen here to develop the finite element model with a least square approach to analyze the effect of liquid flows through a two-step curved converging channel. The different flow phenomena will be established in the shape of streamline patterns, vortex intensity, and vortex length in terms of the different Reynolds numbers.

Momentum Equation

The required solution is a steady state, whereas local acceleration vanishes for the governing equations, such as continuity and momentum equations. Initially, the 2nd-order governing equations are described as follows. Momentum equations in general fluid mechanics, these equations are expressed as Newton's law. It is the expression describing the relationship of the force applied onto the fluid unit and the mass of the fluid in the unit, and the velocity of fluid movement.

Firstly, the governing equations are described as follows;

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

The research problem is chosen here, a two-dimensional mathematical model for the liquid (water), and it may be used to examine the behavior of a Newtonian fluid

through a double backward step channel.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

X – Momentum

$$\frac{\partial u}{\partial t} + \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \frac{1}{\text{Re}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (3)$$

Y – Momentum

$$\frac{\partial v}{\partial t} + \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \frac{1}{\text{Re}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (4)$$

$$U = V_{\text{mean}} \times s \times (1-s) \quad (5)$$

NUMERICAL SCHEME

$$\boldsymbol{w} = \nabla \times \boldsymbol{u} \quad (6)$$

The vorticity substituted into the continuity and momentum equations resulted as under;

$$\nabla \cdot \boldsymbol{u} = 0 \quad (7a)$$

$$\frac{1}{\text{Re}} \nabla \times \boldsymbol{w} + \nabla P + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = 0 \quad (7b)$$

$$\boldsymbol{w} - \nabla \times \boldsymbol{u} = 0 \quad (7c)$$

Here, the research problem is chosen, the two-dimensional mathematical model for the liquid (water), and it may be transported to examine the behaviour of Newtonian through a double backward step channel. Thus, the resulting equations are expressed as follows;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8a)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial P}{\partial x} + \frac{1}{\text{Re}} \frac{\partial w}{\partial y} = 0 \quad (8b)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial P}{\partial y} - \frac{1}{\text{Re}} \frac{\partial w}{\partial x} = 0 \quad (8c)$$

$$w + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \quad (8d)$$

After that, the above system (8c) can be transformed into matrices and develop the coefficient matrices like A_1 , A_2 , A_3 , and A .

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ u & 0 & 1 & 0 \\ 0 & u & 0 & -\frac{1}{Re} \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ v & 0 & 0 & \frac{1}{Re} \\ 0 & v & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} u \\ v \\ p \\ w \end{pmatrix}$$

The matrix system

$$KU=f \quad (9)$$

The domain consists of the different zones, like the stationary walls, the inlet, and the outlet boundaries. Therefore, fixed no-slip boundary conditions are imposed, a parabolic inlet profile is imposed for converge solution, and an extra variable, like vorticity assumed. At the outlet, the pressure vanished, and the velocity components obtained the approximate solution.

Problem Definition and Mathematical

Formulation

The problem focused on the various rectangular channels combined as a series of circular steps, which relates to the backwards step channel. The detailed measurement of the lengths and height of the walls and other zones, like the inlet and outlet, is prescribed in the diagram. Due to the parabolic classification of the equations, a parabolic velocity profile is chosen at the inlet of the channel, and zero pressure is imposed at the outlet of the channel. The detailed statistics of the research domain with details of discretization are Minimum element quality 0.7573, average element quality 0.976, and the height of the triangular elements is 1317, total edge elements are 133, and total vertex elements are 8, respectively. The

number of degrees of freedom is solved for 2178. The predicted results are fully compared with other predicted and experimental results of Ahmed (2015), Omar (2017), (2015).

Mesh statistics	Number of elements
Minimum element equality	0.7573
Average element equality	0.976
Triangular elements	1317
Edge elements	133
Vortex elements	8
Number of degrees of freedom solved for	2171

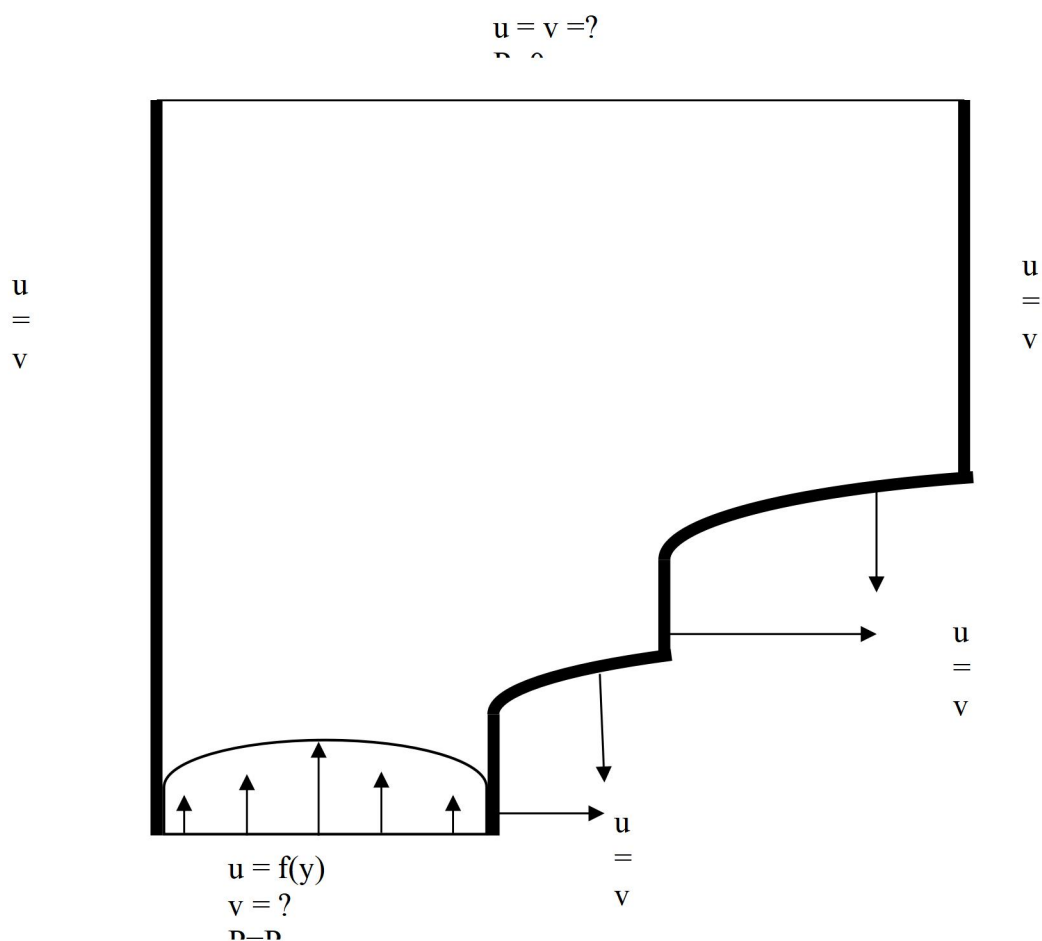


Figure 01: ScA schematic diagram of two steps curved backward channel.

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

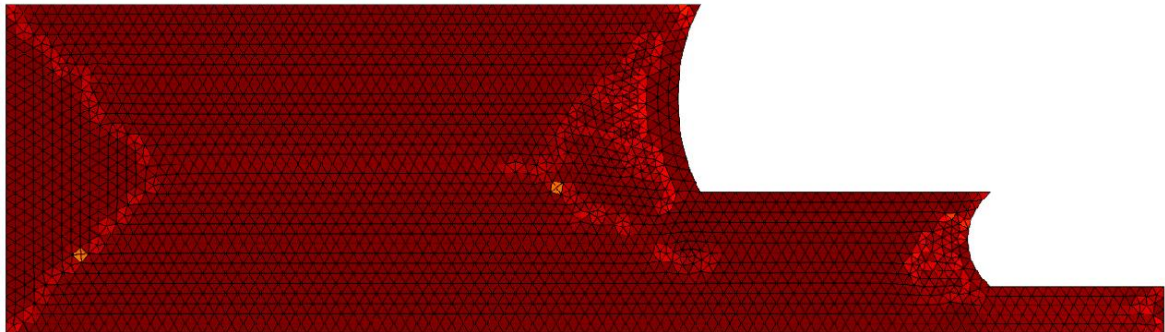
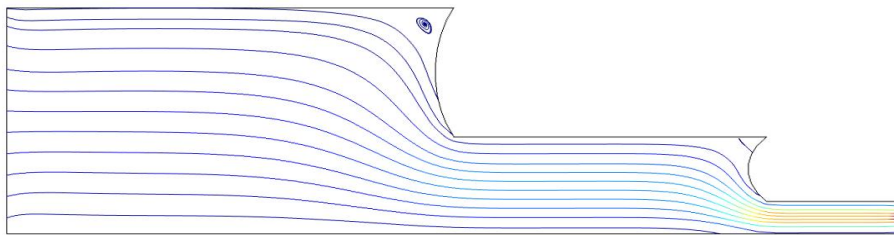


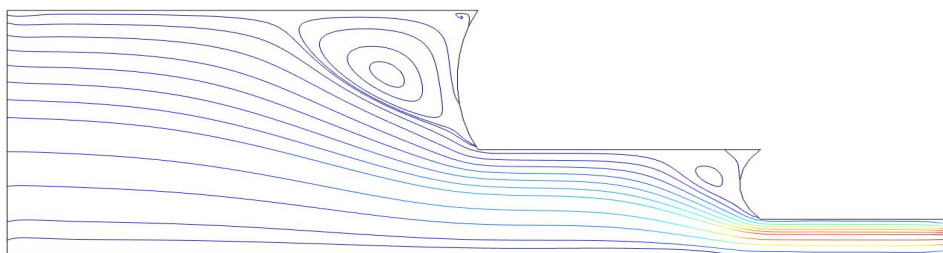
Fig 2: Extremely fine mesh of a through two-step curved converging channel.

Results and Discussion

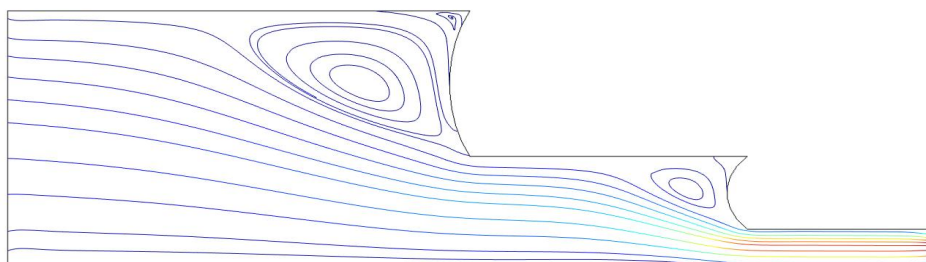
Re=01



Re=500



Re=700



Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

Re=1000

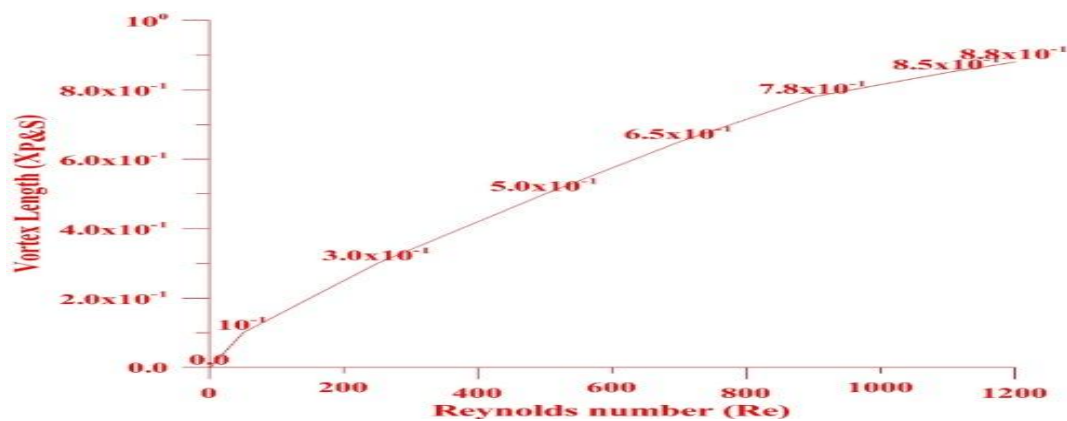
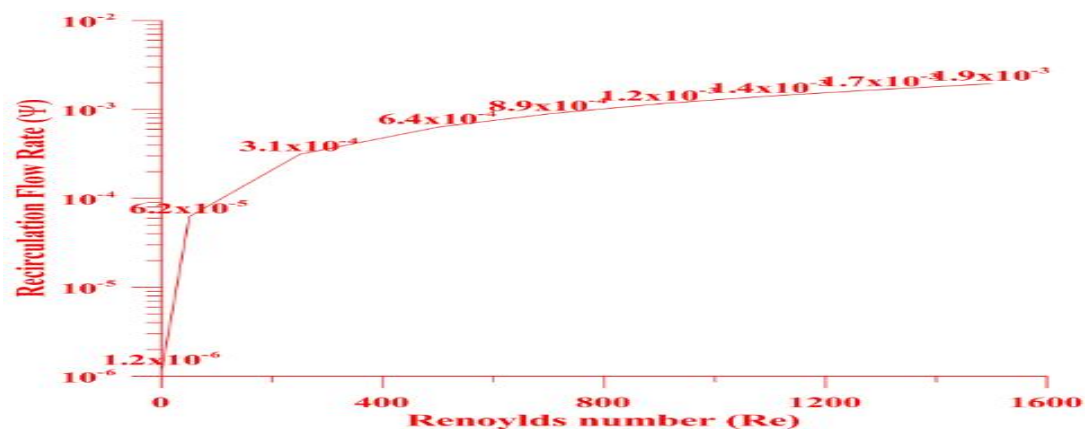
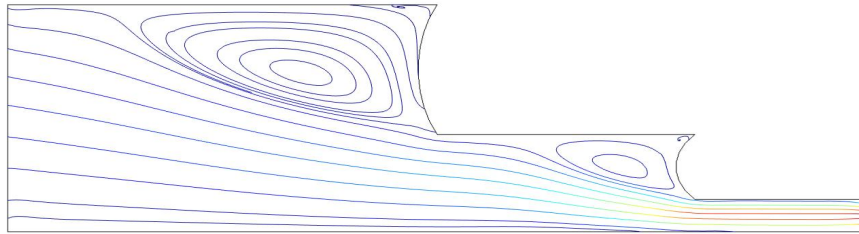


Figure 4: Recirculation flow rate and vortex length

Online ISSN

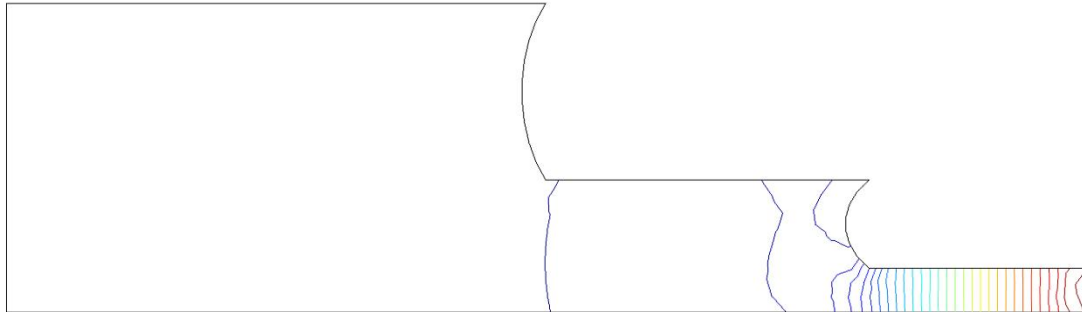
3007-3197

Print ISSN

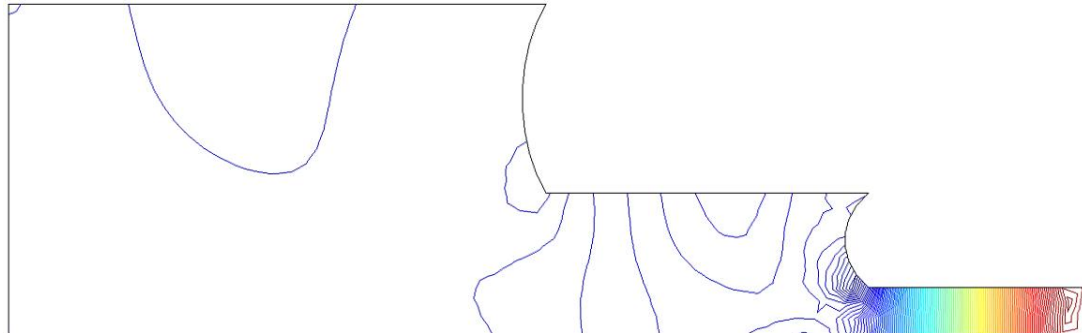
3007-3189

<http://amresearchreview.com/index.php/Journal/about>

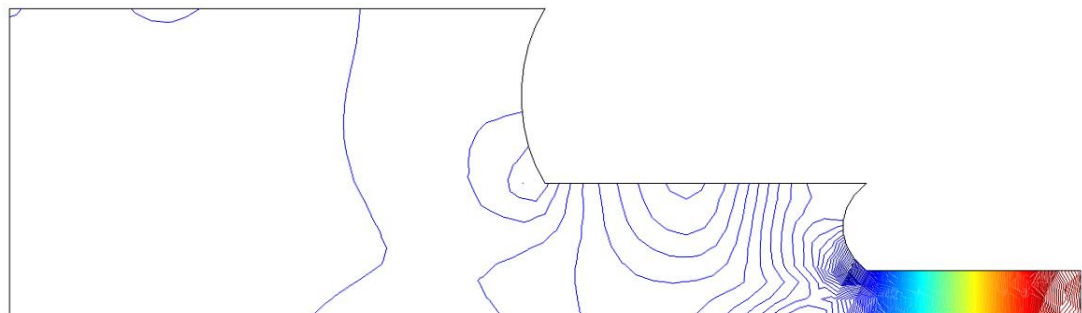
Re=01



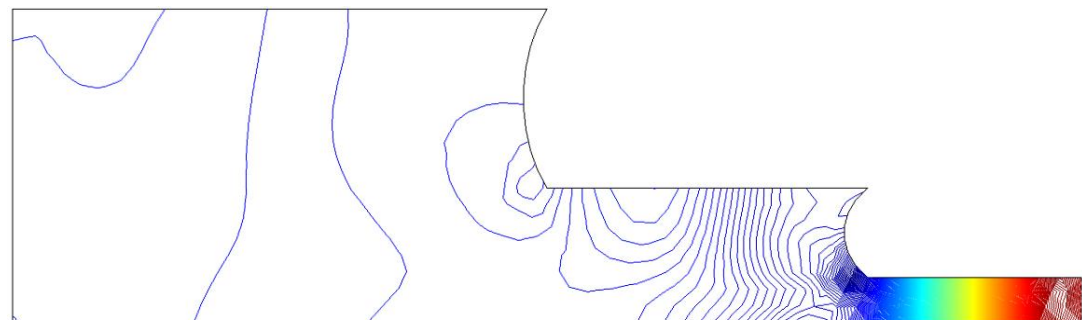
Re=500



Re=700



Re=1000



Recirculation Flow rate(Ψ)		
Reynolds number(Re)	Pressure	
	($\Psi_{1 \min}$)	($\Psi_{1 \max}$)
01	9.71e-9	1.79e-7
250	1.09e-6	1.79e-4
500	1.73e-5	3.35e-4
700	4.87e-5	4.54e-4
850	7.92e-5	5.42e-4
1000	1.15e-4	6.28e-4
1100	1.43e-4	6.84e-4
1200	1.72e-4	7.41e-4
1500	2.75e-4	9.15e-4

Conclusion

At last, it is described that all objectives are successfully achieved, and an approximate solution of the momentum equation for the purpose of the velocity and pressure. The approximate solution preferred for the steady state position is limited to Newtonian and incompressible fluids. For the fluid dynamics, the preferred two circular step channel is the shape available in canals (flumes), rivers, and dams. In the start, the streamline of velocity is plotted to examine the behavior of laminar flows through a curved two-step channel, with testing of different Reynolds numbers. At low Reynolds number (01), the very tiny primary and secondary eddies are observed, and with increasing the Reynolds number, the primary vortex is slowly enhanced, and the secondary eddy quickly grows due to the large area of the domain. At up to 500 Re's, the primary vortex occupied the region and remained stable in the corner of the 1st step, but the secondary vortex continuously increased

up to 1500 and occupied the whole region of the geometry. After the velocity, 2nd parameter, pressure, is also computed at various testing of Reynolds numbers through the curved two-step channel. The pressure contours are increased due to the enhanced Reynolds numbers, and as the Reynolds numbers increase, the excess pressure drops gradually and are visualized through the graph of the excess pressure drop in an increasing pattern. The predicted solution is successfully applicable in the two-step phenomena, but it is also extendable for the 3rd step and more step phenomena.

Acknowledgement:

The research scholar and all coauthors are highly thankful to the concerned department of mathematics, Shah Abdul Latif University, Khairpur.

References:

- Armaly, B.F., et al. (1983), "Experimental and Theoretical Investigation of Backward-Facing Step Flow". J. Fluid Mech., Vol. 127: pp. 473-496.
- Baloch, A (1994), Numerical Simulation of Complex Flows of Non-Newtonian Fluids, Ph.D. Thesis, University of Wales, Swansea, UK.
- Baloch, A., Townsend, P., Webster, M.F., "On Two- and Three-Dimensional Expansion Flows", Compute. Fluids, Vol. 24, pp. 863-882.
- Biswas, G., Breuer, M. and Durst, F., (2004), "Backward-facing Step Flows for Various Expansion Ratios at Low and Moderate Reynolds Numbers", J. Fluids Eng.-Trans. ASME, Vol. 126 (3): p. 362-374.
- c.chafi. A.hazzab. (2010), "Study of Flow and Energy Dissipation in Stepped", Jordan Journal of Civil Engineering, Volume 4, No. 1, 2010 Spillways.
- Herry, B.B., et al., (2011), "Flow Bistability Downstream of Three-Dimensional Double Backward-Facing Steps at Zero Degree Sideslip", J. Fluids Eng.-Trans. ASME, Vol. 5, pp. 133.
- Hussein, et al. (2013), "CFD Simulation of Heat Transfer and Turbulent Fluid Flow over a Double Forward-Facing Step ", Hindawi Publishing Corporation Mathematical Problems in Engineering Vol. 2013, Article ID 895374, 10 pages.

- Khudheyer S. M., (2011), "Simulation of Turbulent Flow and Heat Transfer over a Backward-Facing Step with ribs turbulators", Thermal Science, Vol. 15, No. 1, pp. 245-255.
- Mandal, D. K., Bandyopadhyay, S., and Chakrabarti, S. (2011), "A Numerical Study on the Flow Through A Plane Symmetric Sudden Expansion With A Fence Viewed As A Diffuser", International Journal of Engineering, Science and Technology, Vol. 3, No. 8, 2011, pp. 210-233.
- Moussa Rassai, Sedigheh Rahbar. (2014), "Numerical Flow Model Stepped Spillways to Maximize Energy Dissipation Using FLUENT Software", Vol. 04, Issue 06 (June. 2014), || V5 || PP 17-25.
- Ready, J. N. (2006). An Introduction to the Finite Element Methods. McGraw-Hill Publishing.
- Schafer, F., Breuer, M. and Durst, F., (2009), "The Dynamics of the Transitional Flow over a Backward-Facing Step", J. Fluid Mech., vol. 623: pp. 85-119.
- Shaikh, H (2016), Numerical Simulation of Expansion and Contraction Flow in Channel Filled with Porous Media, Ph.D. Thesis, Shah Abdul Latif University, Khairpur.
- Shaikh, H., Shah, S.B., Solangi, M.A., and Baloch, A. (2012). A Computer Simulation of the Flow of Newtonian Fluid through Backward Step Channel, Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 44 (4) 703-708.
- Sotoodehnia, S., Aminizadeh, N. and Atashafrooz, M. (2016), "Effects of Inclination Angle on the Irreversibility in a Duct with Separated Flow", International Journal of Mechanics and Applications, 6(1): 8-12.
- Stefan Felder, Hubert Chanson. (2016), "Energy Dissipation Down A Stepped Spillway with Nonuniform Step Heights", School of Civil Engineering, the University of Queensland, Brisbane, QLD 4072, Australia.
- Taylor, T. and Ndefo, E. (1971), "Computation of Viscous Flow in A Channel by the Method of Splitting", Proceedings of the Second International Conference on Numerical Methods in Fluid Dynamics, Springer.
- Tinney, C. E. and Ukeiley, L. S. (2009), "A study of a 3-D Double Backward-Facing Step", Exp Fluids Springer-Verlag, vol. 47, pp. 427-438.

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

- Tinney, C. E. and Ukeiley, L. S.(2009), “A study of a 3-D Double Backward-Facing Step”,
- Togun, H. (2016), “Laminar CuO–Water Nano-Fluid Flow and Heat Transfer in A Backward-Facing Step with and Without Obstacle”, ApplNanosci, Springer, Vol. 6, pp. 371–378.
- Wang, S., Burton, D., Sheridan, J. And Thompson, M. C. (2014), Characteristics of Flow over a Double Backward-Facing Step”, 19th Australasian Fluid Mechanics Conference, Melbourne, Australia, pp. 8-11.
- Wong, M. K., Sheng, L. C., Nor Azwadi, C. S., and Hashim, G. A. (2015), “Numerical Study of Turbulent Flow in Pipe with Sudden Expansion”, J. of Adv. Res. in Fluid Mech. and Thermal Sciences, Vol. 6, No. 1. Pages 34-48.
- Zhao Rui. Ron Ji-Li. (2015), “Entropy-Based Detached-Eddy Simulation of the Air Wake Over A Simple Frigate Shape”, Systems Engineering Research Institute, Beijing, China.