

A Pythagorean Fuzzy Set–Based Approach for Identifying the Best City for Living in Pakistan

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ABSTRACT

This work presents and formalizes the idea of Pythagorean fuzzy $\hat{G}$ compact spaces, exploring its structural aspects and distinctive features. The work broadens the theoretical underpinnings of fuzzy topology by utilizing the intrinsic flexibility of Pythagorean fuzzy sets, which have membership and non-membership values squared and summed to a maximum of one. As a practical example of this theory, the study examines its application in assessing and selecting suitable cities in Pakistan based on temperature criteria using real-time data. This application demonstrates the versatility of Pythagorean fuzzy $\hat{G}$ compact spaces in dealing with multi-dimensional and imprecise facts. Temperature fluctuations might have overlapping or gradual impacts on decision-making. The technique allows for the categorization and ranking of cities by properly simulating the ambiguity inherent in natural phenomena such as weather conditions. This is especially important in a nation like Pakistan, where climatic variety is widespread and temperature swings vary greatly between locations. Furthermore, this work sheds light on how Pythagorean fuzzy topological spaces may be used as a useful tool for data-driven decision-making in environmental science, urban planning, and resource allocation. The capacity to incorporate real-world, inaccurate data into a mathematically rigorous framework shows the link between theoretical advances and their practical applications.

This study explores the features of Pythagorean fuzzy $\hat{G}$ compact spaces, laying the groundwork for future research and practical applications in fields where uncertainty and complexity overlap. This work not only broadens our understanding of fuzzy topological structures but also illustrates their use in tackling current issues in data processing and decision-making.

Introduction

Since many classes of things seen in the actual world lack clearly defined membership criteria, Zadeh [1] developed the idea of a fuzzy set. This illustrates the difference between the precise, binary character of traditional mathematical representations based on logic and exact numbers and mental representations of reality. In these representations, membership in a category is represented by "fuzzy predicates" or gradual characteristics. Since then, Zadeh's fuzzy set theory has developed into a basis for managing ambiguity and uncertainty in several fields, such as knowledge representation and artificial intelligence. Possibility theory, a paradigm for thinking about possibility and certainty as progressive modalities, was developed as a result of the gradual membership concept. Atanassov [2] introduced the idea of non-membership. The intuitionistic fuzzy sets were introduced by using the concept of membership and non-membership functions and Coker [3] studied its properties. By the time fuzzy sets developed, Yager and his associates [4] created an extension of the IFS called the Pythagorean fuzzy set (PFS).

Compactness plays a fundamental role across various branches of mathematics. The application of Pythagorean fuzzy compactness in decision-making is particularly noteworthy. In this paper, we present several properties of Pythagorean fuzzy compact spaces and identify the preferred city in Pakistan using a Pythagorean fuzzy GGG-compact space, considering temperature as the key parameter. The primary objective of this study is to introduce a topological framework for Pythagorean Fuzzy Sets (PFS). Our motivation lies in exploring the effectiveness of the Pythagorean fuzzy GGG-compact space in addressing the problem of selecting the most suitable cities to live in Pakistan through the maxima method, given its broad applicability to real-life situations characterized by uncertainty.

Overall, the study aims to examine the concept of Pythagorean fuzzy GGG-compactness and demonstrate its usefulness in analyzing real-time, temperature-based data.

Preliminaries

This section contains definitions of some basic concepts, their abbreviations and symbols are also discussed.

Notations:

Pythagorean fuzzy $\hat{G}$ closed set: **PF $\hat{G}$ Cs** Pythagorean fuzzy $\hat{G}$ interiors: **PF $\hat{G}$ -int**
Pythagorean fuzzy set : $\square$ **Fs** Pythagorean fuzzy $\hat{G}$ closure: **PF $\hat{G}$ -clo**
Pythagorean fuzzy topological space: **PF-TSP** Pythagorean Fuzzy $\hat{G}$ open cover: **PF $\hat{G}$ -OC** Pythagorean fuzzy open set : **PFOs** Pythagorean Fuzzy open cover: **PF-OC** Pythagorean fuzzy closed set: **PFCs** Pythagorean Fuzzy Compact: **PF-Co** Pythagorean fuzzy interiors: **PF-int**
Pythagorean Fuzzy $\hat{G}$ Compact space: **PF $\hat{G}$ -CoS** Pythagorean fuzzy closure: **PF-clo** Finite intersection properties: **F-IP** Pythagorean fuzzy $\hat{G}$ open set: **PF $\hat{G}$ Os** Pythagorean fuzzy $\hat{G}$ open set: **PF $\hat{G}$ Os** Pythagorean fuzzy $\hat{G}$ closed set: **PF $\hat{G}$ Cs** Pythagorean fuzzy topology: **PF-T** Pythagorean fuzzy $\hat{G}$ compact: **PF $\hat{G}$ -Co**

Definition 2.1: [2] Let X be a nonempty set. A mapping $F: X \rightarrow [0, 1]$ such that $F = (\nu_F, \omega_F)$ and $\nu_F + \omega_F \leq 1, \forall x \in X$. Where ν_F and ω_F are membership and nonmembership values respectively. Then F is said to be an Intuitionistic fuzzy set.

Definition 2.2: [2] Let X be a non-empty set then a Pythagorean fuzzy set is defined as

$$F = \{f, \nu_F(f), \omega_F(f) : f \in X, \nu_F(f): X \rightarrow [0,1] \text{ and } \omega_F(f): X \rightarrow [0,1] \text{ such that } 0 \leq \nu_F(f)^2 + \omega_F(f)^2 \leq 1\}$$

Example.2.3. Let $X = \{l, m, n, o, p\}$ be any crisp collection then one can describe the PFs as

$$F = \{(l, 0.0008, 0.001), (m, 0.1, 0.04), (n, 0.05, 0.2), (o, 0.0007, 0.000001), (p, 0.5, 0.8)\}$$

Definition 2.4: [2] Let $M = (\nu_m, \omega_m)$ and $N = (\nu_n, \omega_n)$ be the two PFs of Z , then

- I. $M \cup N = (\max(\nu_m, \nu_n), \min(\omega_m, \omega_n))$
- II. $M \cap N = (\min(\nu_m, \nu_n), \max(\omega_m, \omega_n))$
- III. $N^c = (\nu_n, \omega_n)$
- IV. $N \subset M$ or $M \supset N$ or if $\nu_n \leq \nu_m$ and $\omega_n \geq \omega_m$.

Definition 2.5: [7] let $N = (\nu_n, \omega_n)$ be a PFs of the Pythagorean fuzzy topological space Z Consequently, PF-int and PF-clo are defined as

- I. $\text{Int}(N) = \bigcup \{G \mid G \text{ is a PFOs in } Z \text{ and } G \subseteq N\}$
- II. $\text{Cl}(N) = \bigcap \{K \mid K \text{ is a PFCs in } Z \text{ and } N \subseteq K\}$

Definition 2.6: [9] A PFs N is said to be a Pythagorean fuzzy regular open set iff $N = \text{int}(\text{cl}(N))$ and a PFs M is said to be a Pythagorean regular closed set iff $M = \text{cl}(\text{int}(M))$.

Definition 2.7: [9] The PFs U of a PF-TS Z is said to be a PFGCs if $\text{cl}(U) \subseteq R$ where R is a PFOs and $R \neq 0_x$ or 1_x . The counterpart to the PFGCs is the PFGOs.

Notation. $\hat{G}_x$ indicates the assortment of all PFGCS in Z .

Definition 2.8 [9] The PFs U of a PF-TS Z is called PFGCs if $\text{cl}(U) \subseteq R$ whenever $U \subseteq R$ where R is a PFOs and $R \neq 0_x$ or 1_x . The counterpart to the PFGCs is the PFGOs.

Definition 2.9 [10] Let Z be a PF-TS and Z and $N = (\nu_n, \omega_n)$ be the PFs in Z . Then, PFG-int and PFG-clo of H are defined as

- I. $\text{int}_{\hat{G}}(H) = \bigcup \{N \mid N \text{ is a PFOs in } Z \text{ and } N \subseteq H\}$
- II. $\text{cl}_{\hat{G}}(H) = \bigcup \{M \mid M \text{ is a PFCs in } Z \text{ and } H \subseteq M\}$

Definition. 2.7 [6]. Let R be a topological space with a collection of open sets whose union contains R , then the collection is known as the open cover of R .

Definition. 2.11 [6]. let (R, τ) be a fuzzy topological space, then a group $\mathcal{E}$ of a fuzzy set is called the open cover of a fuzzy set F iff $F \subset \bigcup \{A | A \in \mathcal{E}\}$ and a fuzzy topological space is compact if and only if each open cover of the space has a finite sub cover.

Definition. 2.12 [9]. Let Z be a nonempty set and the group of Pythagorean fuzzy subsets of Z represented as $\mathfrak{t}$ fulfills the following conditions.

- I. $0_x, 1_x \in \mathfrak{t}$.
- II. $L, M \in \mathfrak{t} \Rightarrow L \cap M \in \mathfrak{t}$.
- III. $L_j, j \in J \in \mathfrak{t} \Rightarrow \bigcup A_j \in \mathfrak{t}$.

Then, $\mathfrak{t}$ is said to be a PF-T for Z , and $(Z, \mathfrak{t})$ is PF-TS.

Definition .2.13 [7]. Let Z be a Pythagorean fuzzy topological space with a collection of open sets whose union contains Z , then the collection is known as the Pythagorean fuzzy open cover .

Definition .2.14 [7]. Let $(Z, \mathfrak{t})$ be a PF-TS and F be a PFs over Z Then, the Pythagorean fuzzy interior and Pythagorean fuzzy closure are defined as

- I. $\text{int}(F) = \bigcup \{H : H \text{ is a PFOs in } Z \text{ and } H \subseteq F\}$,
- II. $\text{cl}(F) = \bigcap \{R : R \text{ is a PFCs in } Z \text{ and } F \subseteq R\}$

Definition 2.15 [11]. PFs from the main data formed by using the Pythagorean fuzzy trapezoidal function, defined as

$$\mathfrak{v}_{PF(x)} = \begin{cases} 0, x \leq q_1, \\ \sqrt{\frac{x-q_1}{q_2-q_1}}, q_1 < x \leq q_2, \\ 1, q_1 \leq x < q_3, \\ \sqrt{\frac{q_4-x}{q_4-q_3}}, q_3 \leq x < q_4, \\ 0, x \geq q_4, \end{cases} \quad \mathfrak{w}_{PF(x)} = \begin{cases} 0, x \leq q_1, \\ 1 - \sqrt{\frac{x-q_1}{q_2-q_1}}, q_1 < x \leq q_2, \\ 1, q_1 \leq x \leq q_3, \\ 1 - \sqrt{\frac{q_4-x}{q_4-q_3}}, q_3 \leq x < q_4, \\ 0, x \geq q_4, \end{cases}$$

Where, $\mathfrak{v}_{PF(x)}$ and $\mathfrak{w}_{PF(x)}$ are membership and nonmember ship functions respectively. q_1, q_2, q_3, q_4

Definition .2.8 [12]. The mean of the maxima method is one of the defuzzification methods that we use in this paper and it can be defined as

$$Q_{\text{MOM}} = \frac{\sum_{i=1}^N Q_i^n}{2N} + \frac{\sum_{i=1}^L Q_i^l}{2L}$$

where $Q^n = \{Q | \mathfrak{v}_Q^n = \text{Max}_{i=1}^n \mathfrak{v}_Q^i\}$ and $Q^l = \{Q | \mathfrak{w}_Q^l = \text{Max}_{i=1}^L \mathfrak{w}_Q^i\}$

Main Results

Pythagorean fuzzy $\hat{G}$ Compact Space (PF $\hat{G}$ CS)

Characteristics of Pythagorean fuzzy $\hat{G}$ compact space will be described in this portion.

Definition 3.1. Let Z be the PF-TS. A collection $\beta = (\forall \beta, \omega_\beta)$ of subsets of Z is called $\hat{G}$ cover of Z .

If the union of the element of β equals to 1_x .

Definition 3.2. Let Z be the PF-TS, a $\hat{G}$ sub cover of a $\hat{G}$ cover, $\beta \neq 0_x$ is a sub-collection of β which is a $\hat{G}$ cover.

Definition 3.3. Let Z be the PF-TS. A $\hat{G}$ cover of β of a PF-TS Z is called Pythagorean fuzzy Z open cover if elements of β are PFOs of Z .

Definition 3.4. let Z be the PF-TS. Then, Z is known as a Pythagorean fuzzy $\hat{G}$ compact space iff each $\text{PF}\hat{G}\text{OC}$ of Z has a finite $\text{PF}\hat{G}$ open sub-cover.

Example 3.5. Suppose $Z = z$ be a PF-TS with topology

$$\tau = \{(0,1), (0.1,0.9), (0.2,0.9), (0.4,0.8), (0.6,0.7), (0.7,0.6), (1,0)\}$$

and

$$\gamma = \{(0.4,0.8), (0.6,0.7), (0.7,0.6), (1,0)\}$$

be the $\hat{G}$ cover of Z and the sub collection $\{(0.6,0.7), (0.7,0.6), (1,0)\}$ is the $\hat{G}$ sub, which is a Pythagorean fuzzy $\hat{G}$ open cover. Hence is a Pythagorean fuzzy $\hat{G}$ compact space.

Proposition 3.6. Every $\text{PF}\hat{G}$ Cs of $\text{PF}\hat{G}$ -CoS Z is $\text{PF}\hat{G}$ -Co.

Proof: let F denote a $\text{PF}\hat{G}$ Cs of Z and let $\{E_j | j \in L\} = \gamma$ be a $\text{PF}\hat{G}$ -OC of Z . As N^c is a $\text{PF}\hat{G}$ Os of Z and $\gamma = \{E_j | j \in L\} \cap F^c$ is $\text{PF}\hat{G}$ OC of Z . As Z is $\text{PF}\hat{G}$ Co, there exists a finite subset L_0 of L such that $\bigcap \{E_j | j \in L\} \cap F^c = L_x$. Hence, F is a $\text{PF}\hat{G}$ -OC.

Proposition 3.7. Every $\text{PF}\hat{G}$ -clo image of $\text{PF}\hat{G}$ -CoS Z is a $\text{PF}\hat{G}$ -Co.

Proof: Let $\Psi: Z \rightarrow M$ be the $\text{PF}\hat{G}$ -clo from $\text{PF}\hat{G}$ -CoS Z onto $\text{PF}\hat{G}$ -CoS M . let $\{E_j | j \in L\}$ be a $\text{PF}\hat{G}$ -OC of M , and let $\{\Psi^{-1}(E_j | j \in L)\}$ be a $\text{PF}\hat{G}$ -OC of because Z is $\text{PF}\hat{G}$ -CoS, it has a finite Pythagorean fuzzy sub cover say $\{\Psi^{-1}E_1, \dots, \Psi^{-1}E_n\}$. Because Ψ on $E_1, \dots, E_n$ is a $\text{PF}\hat{G}$ -Co of Z , M is a $\text{PF}\hat{G}$ -CoS.

Proposition 3.8. A PF-TS Z is a PF-Co if for every collection $\{E_j | j \in L\}$ of PFCs of Z with F-IP $M \cap E_j \neq 0_x$.

Proof: Let $\{E_j | j \in L\}$ be a collection of PFCs in Z with F-IP. Assume that $\bigcap E_j = 0_x$. Then, $\bigcup E_j^c = 0_x$. Because Z is a PF-Co, hence there exists $j_1, j_2, \dots, j_n$ such that $\bigcup E_{j_k}^c \neq 1_x$, $\bigcap E_{j_k} = 0_x$ which is a contradiction. Hence, conversely, let $\{E_j | j \in L\}$ be the PF-OC of Z . suppose that for every finite $j_1, j_2, \dots, j_n$ we have that $\bigcap E_{j_k} \neq 1 - x$ and $\bigcap E_{j_k}^c \neq 0_x$. Hence, it is F-IP. The hypothesis $E_j^c \neq 0_x$ implies that $\bigcup E_{j_k}^c \neq 1_x$, this contradicts $\{E_j | j \in L\}$ is a PF-OC at This, Z is PF-Co.

Identification of Preferred Place to Lead a Better Life through Pythagorean Fuzzy $\hat{G}$ Compact Space:

In this section, we will describe the application of a Pythagorean fuzzy $\hat{G}$ compact space to find a suitable city according to temperature to live in Pakistan. The degree of warmth or coldness of the body or environment is known as Temperature. The body temperature of living things can easily be affected by humidity and temperature is the main factor in the Earth's climate. The climate in a place will be cooler if the temperature is low, whereas the climate will be warmer if the temperature is hot. Our day-to-day is completely affected by the degree of temperature. We have to consider temperature to decide what to wear, what to cook, where to go, and in many other situations. Temperature checking is essential while deciding whether we can ever go out or not. Hence, temperature is supposed as an important parameter for living in a specific place or city. The temperatures of different cities were taken from 1995 to 2018 and the trapezoidal membership function was used for assigning membership and non-membership values to the temperature values. Pythagorean fuzzy $\hat{G}$ closed sets are formed. Then cover and sub cover are derived from the given data and a Pythagorean fuzzy $\hat{G}$ compact was framed.

Algorithm

Step 1. The average temperature of 12 months for the four big cities of Pakistan i.e.; Peshawar, Mumbai, Karachi, and Kolkata from the year 1995-2018 has been collected.

Step 2. Trapezoidal membership functions were used to form Pythagorean fuzzy sets for each year by assigning $(\nu_{PF(x)}, \omega_{PF(x)})$ to the corresponding temperature value.

Step 3. The obtained Pythagorean fuzzy set fulfills all the conditions that is given in the definition of Pythagorean fuzzy $\hat{G}$ closed sets and the graph of the Pythagorean fuzzy sets is drawn by MATLAB.(figure 2)

Step 4. $\hat{G}$ Cover of the Pythagorean Fuzzy topological space is formed.

Step 5. Pythagorean fuzzy $\hat{G}$ compact space is obtained.

Step 6. The mean of maxima method is used for defuzzification and preference is given according to the obtained value

$$Q_{MOM} = \frac{\sum_{i=1}^N Q_i^n}{2N} + \frac{\sum_{i=1}^L Q_i^l}{2L},$$

$$\text{Where } Q^n = \{Q | \nu_{Q^n} = \text{Max}_{i=1}^n \nu_{Q_i}\} \text{ and } Q^l = \{Q | \omega_{Q^l} = \text{Max}_{i=1}^L \omega_{Q_i}\}$$

Step 7. Preferences were assigned to the cities according to the obtained value using a decision-making technique.

Detailed study of the Proposed Algorithm to Find an Optimal City

Step 1. Through an online platform temperature data from 1995-2018(January to

December) were collected for the main and famous cities of Pakistan; Peshawar, Karachi, Kolkata, and Mumbai. For each year, the average of 12 months was calculated. So, the temperature averages in Karachi, Mumbai, Kolkata, and Peshawar were obtained.

In Table 1, the average temperatures of Karachi, Peshawar, Mumbai, and Kolkata are shown in columns 1, 4, 7 and 7 respectively. The membership and nonmember ship values obtained by the Trapezoidal membership function for the cities; Karachi, Peshawar, Mumbai, and Kolkata were shown in columns 2, 3, 5, 6, 8, 9, 11, and 12 respectively.

Step 2. The Pythagorean fuzzy trapezoidal function was used to find Pythagorean fuzzy sets by setting $q_1 = 78.9$, $q_2 = 81$, $q_3 = 83$, $q_4 = 83.5$

$$\mu_{PF(x)} = \begin{cases} 0, x \leq q_1, \\ \sqrt{\frac{x-q_1}{q_2-q_1}}, q_1 < x \leq q_2, \\ 1, q_1 \leq x < q_3, \\ \sqrt{\frac{q_4-x}{q_4-q_3}}, q_3 \leq x < q_4, \\ 0, x \geq q_4, \end{cases} \quad \eta_{PF(x)} = \begin{cases} 0, x \leq q_1, \\ 1 - \sqrt{\frac{x-q_1}{q_2-q_1}}, q_1 < x \leq q_2, \\ 1, q_1 \leq x \leq q_3, \\ 1 - \sqrt{\frac{q_4-x}{q_4-q_3}}, q_3 \leq x < q_4, \\ 0, x \geq q_4, \end{cases}$$

Where $\mu_{PF(x)}$ and $\eta_{PF(x)}$ are membership (μ) and nonmember ship (η) functions respectively. By using MATLAB (version R2021a), a Pythagorean fuzzy set graph was created and each point on the graph shows the PFs and the associated nonmember ship and membership values of that temperature.

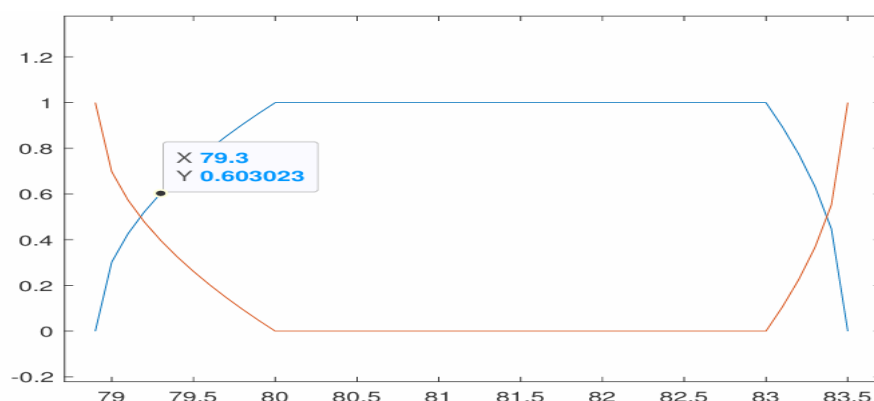


Figure1. Representation of fuzzified value of Pythagorean fuzzy set using MATLAB.

Example: Here we do calculations taking single average temperature values of each city using the trapezoidal membership and nonmember ship function.

For $x = 82.7156$

As x is in between q_1 and q_3 , then the $\mu_{PF(x)}$ and $\eta_{PF(x)}$ are

$\forall \text{PF}(x) (82.7156) = 1$ and $\exists \text{PF}(x) = 0$

For $x = 76.9983 \leq q_4$ the membership and non membership values are

$\forall \text{PF}(x) = 0$ and $\exists \text{PF}(x) = 1$

Karachi	M	η	Peshawar	M	η	Quetta	M	η	Lahore	M	η
82.7156	1	0	76.9983	0	1	79.467	0.59305	0.8058	81.746	1	0
82.6576	1	0	75.7541	0	1	79.529	0.6238	0.7815	81.7451	1	0
83.3838	0.4820	0.8762	74.4573	0	1	78.7301	0	1	81.7858	1	0
84.2123	0	1	76.7767	0	1	79.9644	0.8156	0.5786	82.1979	1	0
83.178	0.6359	0.7718	77.1373	0	1	79.8025	0.757	0.6603	81.8214	1	0
82.9820	1	0	76.277	0	1	79.7008	0.7075	0.7067	81.774	1	0
83.2123	0.7585	0.6517	76.277	0	1	78.9021	0.0370	0.9993	81.0630	1	0
83.4660	0.907	0.9654	77.6841	0	1	80.2411	0.9155	0.4022	82.751	1	0
83.4921	0.191	0.9920	75.8663	0	1	79.896	0.767	0.6488	81.4370	1	0
82.5467	1	0	77.787	0	1	80.3885	0.9645	0.940	80.607	1	0
82.7312	1	0	76.5595	0	1	79.1142	0.3659	0.9306	81.925	1	0
82.9647	1	0	77.7984	0	1	79.9589	0.8135	0.5815	81.3005	1	0
82.0337	1	0	77.0748	0	1	79.5756	0.6498	0.7601	82.5531	1	0
82.3557	1	0	76.956	0	1	79.724	0.4824	0.8759	82.1156	1	0
84.3123	0	1	77.8814	0	1	80.4962	0.9988	0.0490	83.0447	0.9543	0.1119
82.9195	1	0	82.2364	0	1	80.567	1	0	82.6871	1	0
82.9373	1	0	76.6962	0	1	79.4178	0.5689	0.8224	82.1147	1	0
83.8896	0	1	76.9049	0	1	80.1792	0.8942	0.4477	81.6964	1	0
82.9622	1	0	76.7625	0	1	79.3855	0.5508	0.8346	81.7392	1	0
83.487	0.1858	0.989	77.0011	0	1	79.8811	0.7831	0.6219	82.7939	1	0
84.1790	0	1	77.6071	0	1	80.6566	1	0	83.6986	0	1
84.5000	0	1	80.5456	1	0	81.3205	1	0	82.9213	1	0
84.7586	0	1	78.9932	0.2413	0.9705	79.8584	0.7739	0.6333	83.4044	0.4373	0.8993
84.9737	0	1	78.3545	0	1	79.7614	0.7337	0.6794	83.88	0	1

For $x = 79.467$ which is in between q_1 and q_2 the membership and non membership values are,

$$\mu_{PF}(x) = \sqrt{\frac{x-q_1}{q_2-q_1}} = \sqrt{\frac{79.467-78.9}{81-78.9}} = 0.64094$$

and

$$\eta_{PF}(x) = 1 - \sqrt{\frac{79.467-78.9}{81-78.9}} = 0.35905$$

Also $x = 81.746 \leq q_1$ the membership and non membership values are,

$$\mu_{PF}(x) = 1 \text{ and } \eta_{PF}(x) = 0$$

Similarly, for each average temperature values were collected (Table 1).

Step 3. We can get the $\hat{PFGCS}$ between the obtained Pythagorean fuzzy sets for the cities Karachi, Quetta, and Lahore, for example for Karachi city it is possible to find the $\hat{PFGCS}$ between the Pythagorean fuzzy sets $(0.7568513, 0.65858)$, $(0.635912, 0.771762)$, $(0.635912, 0.771762)$ and $(0.482005, 0.87688)$.

Step 4. $\hat{G}$ cover of the cities Quetta, Karachi, and Lahore is obtained from their $\hat{PFGCS}$ but there is no $\hat{G}$ cover for Peshawar because it did not have any $\hat{PFGCS}$.

Step 5. Hence, Pythagorean fuzzy $\hat{G}$ compact space is formed for the cities excluding Peshawar city because there is no $\hat{G}$ cover for Peshawar.

Table.1. Average temperature values with membership and nonmember ship values corresponding to each city from the year 1998-2018.

Step 6. The mean of maxima method for PFs fuzzy-rule-based systems uses fuzzification, inference, and composition processes to evaluate linguistic if-then rules. They produce fuzzy results that must be converted into crisp outputs. Defuzzification was used to transform the fuzzy findings into crisp findings. The technique of converting a fuzzy output into a unique crisp value for a fuzzy set under consideration is known as defuzzification. There are many Defuzzification techniques in the decision-making process, Mean of the Maxima method is used due to its effective and suitable results for this real-time data.

A Pythagorean fuzzy set was defined and used to find preferences for the city according to the average temperature through the mean of the maxima method. For a PFs, it is given as

$$Q_{MOM} = \frac{\sum_{i=1}^N q_i^n}{2N} + \frac{\sum_{i=1}^L q_i^l}{2L}$$

Where $Q^n = \{Q | \mu_{Q^n} = \max_{i=1}^n \mu_{Q_i}\}$ and $Q^l = \{Q | \eta_{Q^l} = \max_{i=1}^L \eta_{Q_i}\}$

Step 7. By using the above formulation, Q_{MOM} is computed for the four cities, for Karachi we obtained 11 values by using the maximum membership value (M) and minimum non-membership value (η). Using the average for all the values, we found 82.70980. In addition, the preferences have been set for other cities (Table 2). According to the single parameter temperature Karachi is the most preferable and suitable city to live in among the four cities; Peshawar, Lahore, and Quetta. The

Mamdani fuzzy inference system (MFIS), with a relatively simple structure that gives suitable results. MFIS was used to compare the above results for a single parameter temperature value though the results by mean of maxima method are flawless can also validate this result of preferences. To examine that Karachi is the most preferred city to live in, the Mamdani fuzzy inference system was used for every average temperature values of the cities Peshawar, Quetta, Karachi, and Lahore.

Table 2. Computed Mean of Maxima values (Q_{MOM}) for the four cities

City	Computed	Preferences
Karachi	82.70980	1 st
Peshawar	80.54563	4 th
Lahore	80.84659	3 rd
Quetta	81.86986	2 nd

Conclusion

Although urbanization influences temperature, the findings show that multiple factors contribute to temperature variation. One significant factor associated with rising pollution levels is temperature itself. From this study, it can be concluded that both pollution and urbanization are major contributors to the increase in temperature. The Air Quality Index (AQI) of Peshawar has deteriorated, leading to higher pollution, which in turn elevates the temperature. Based on the available data and sources, Peshawar emerges as the least desirable city to live in, primarily due to temperature-related concerns.

Using the QMOM approach, the results indicate that Karachi is the most preferred city in terms of temperature, whereas Peshawar ranks the lowest. This may be attributed to its comparatively higher pollution levels, which contribute to increased temperature and hotter weather conditions.

To validate these results, the Mamdani FIS was applied. This method confirms the relationship between the variables without modifying the input or output functions. The main purpose of the FIS analysis was to evaluate the findings using a topological perspective. The overall assessment clearly shows that Peshawar is not as favorable a city to live in compared to the others.

Table 3. Preferences

City	Preferences
Karachi	First
Quetta	Second
Lahore	Third
Peshawar	Four

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